GLOBAL EDITION

Using and Understanding Mathematics

A Quantitative Reasoning Approach

SIXTH EDITION

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ALWAYS LEARNING

PEARSON

Math for College, Career, and Life

We use math in our day-to-day lives even when we don't realize it. The goal of this book is to increase mathematical literacy so we use it more effectively in everyday life. Mathematics can help us to understand a variety of topics and issues, making us more aware of both the uses and abuses of math. The ultimate goal is to become better educated citizens and be successful in our college experiences, our careers, and our lives.

Each chapter offers an **Activity** designed to spur discussion of some interesting facet of the topics covered in the chapter. [p. 314, 5A]

ACTIVITY

Cell Phones and Driving

Use this activity to gain a sense of the kinds of problems this chapter will enable you to study.

Is it safe to use a cell phone while driving? The science of statistics provides a way to approach this question, and the results of many studies indicate that the answer is no. The National Safety Council estimates that approximately 1.6 million car crashes each year (more than a quarter of the total) are caused by some type of distraction, most commonly the use of a cell phone for talking or texting. In fact, some studies suggest that merely talking on a cell phone makes you as dangerous as a drunk driver. As preparation for your study of statistics in this chapter, work individually or in groups to research the issues raised in the following questions. Discuss your findings.

IN YOUR WORLD

Web Searches to Verify Web Sources

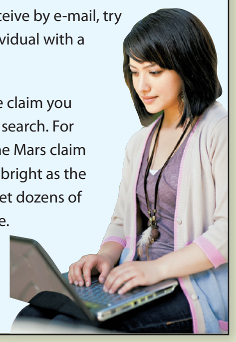
While some information on the Web is inaccurate or biased, the Web is also a great source for checking the accuracy of information. A good way to start is with "fact checking" websites, as long as you also verify that the fact checkers have a reputation for fairness and accuracy. A few reputable fact-checking sites include:

…rg, supported by the non-…lic Policy Center; PolitiFact. …he Fact Checker," a blog …e.

…dd claims, Snopes.com

- To check the validity of messages you receive by e-mail, try TruthOrFiction.com, run by a private individual with a reputation for fairness and accuracy.

If none of those sources has covered the claim you are investigating, try a plain language Web search. For example, if you type the first sentence of the Mars claim ("On August 27, Mars will look as large and bright as the full Moon . . .") into a search engine, you'll get dozens of hits that discuss the claim and why it is false. Of course, if your search turns up conflicting claims about accuracy, you'll still need to decide which claims to believe.



IN YOUR WORLD

47. **Political Action.** This unit outlined numerous budgetary problems facing the U.S. government, as they stood at the time the book was written. Has there been any significant political action to deal with any of these problems? Learn what, if anything, has changed over the past couple of years, then write a one-page position paper outlining your own recommendations for the future.
48. **Debt Problem.** How serious a problem is the gross debt? Find arguments on both sides of this question. Summarize the arguments, and state your own opinion.

In Your World boxes focus on topics that students are likely to encounter in the world around them, whether in the news, in consumer decisions, or in political discussions. This is further enhanced with In Your World exercises, designed to spur additional research or discussion that will help students relate the unit content to the themes of college, careers, and life. [p. 309, 4F and p. 39, 1A,]

Does It Make Sense? questions test conceptual understanding by asking students to decide whether the given statements are sensible and to explain why or why not. These questions encourage students to stop and think critically about a problem rather than just focusing on getting an answer. [p. 380, 5E]

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. There is a strong negative correlation between the price of tickets and the number of tickets sold. This suggests that if we want to sell a lot of tickets, we should lower the price.
8. There is a strong positive correlation between the amount of time spent studying and grades in mathematics classes. This suggests that if you want to get a good grade, you should spend more time studying.

USING TECHNOLOGY

The Compound Interest Formula

Standard Calculators You can do compound interest calculations on any calculator that has a key for raising numbers to powers (y^x or ∧). The only "trick" is making sure you follow the standard **order of operations:**

1. Parentheses: Do terms in parentheses first.
2. Exponents: Do powers and roots next.
3. Multiplication and Division: Work from left to right.
4. Addition and Subtraction: Work from left to right.

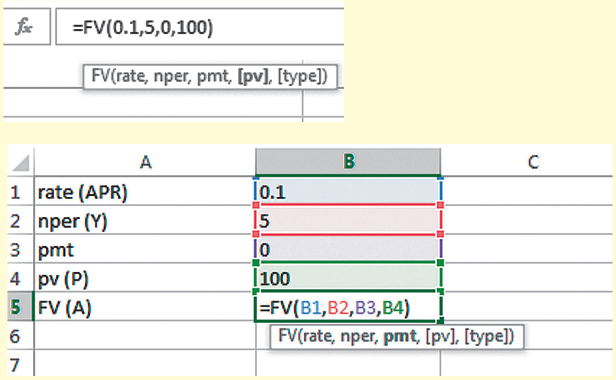
You can remember the order of operations with the mnemonic "Please Excuse My Dear Aunt Sally."

Let's apply this order of operations to the compound interest problem from Example 2, in which we have $P = \$100$, APR $= 0.1$, and $Y = 5$ years.

General Procedure	Our Example	Calculator Steps	Output
$A = P \times (1 + \text{APR})^Y$ 1. parentheses 2. exponent 3. multiply	$A = 100 \times (1 + 0.1)^5$ 1. parentheses 2. exponent 3. multiply	Step 1 1 + 0.1 =	1.1
		Step 2 ∧ 5 =	1.61051
		Step 3 × 100 =	161.051

Note: Do not round answers in intermediate steps; only the final answer should be rounded to the nearest cent.

Excel Use the built-in function FV (for *future value*) for compound interest calculations in Excel. The screen shot to the right shows the use of this function for our sample calculation. The table at the bottom explains the inputs that go in the parentheses of the FV function. Note: You could get the final result by typing values directly into the FV function, but as shown in the screen shot, it is better to show your work. Here we put variable names in Column A and values in Column B, using the FV function in cell B5. Besides making your work clearer, this approach makes it easy to do "what if" scenarios, such as changing the interest rate or number of years.



Input	Description	Our Example
rate	The interest rate for each compounding period	Because we are using interest compounded once a year, the interest rate is the annual rate, APR $= 0.1$.
nper	The total number of compounding periods	For interest compounded once a year, the total number of compounding periods is the number of years, $Y = 5$.
pmt	The amount of any payment made each month	No payment is being made monthly in our example, so we enter 0.
pv	The present value, equivalent to the starting principal P	We use the starting principal, $P = 100$.
type	An optional input related to whether monthly payments are made at the beginning (type $= 0$) or end (type $= 1$) of a month	Type does not apply in this case because there is no monthly payment, so we do not include it.

Compound Interest as Exponential Growth

The New College case demonstrates the remarkable way in which money can grow with compound interest. Figure 4.2 shows how the value of the New College debt rises during the first 100 years, assuming a starting value of \$224 and an interest rate of 4% per year. Note that while the value rises slowly at first, it rapidly accelerates, so in later years the value grows by much more each year than it did during earlier years.

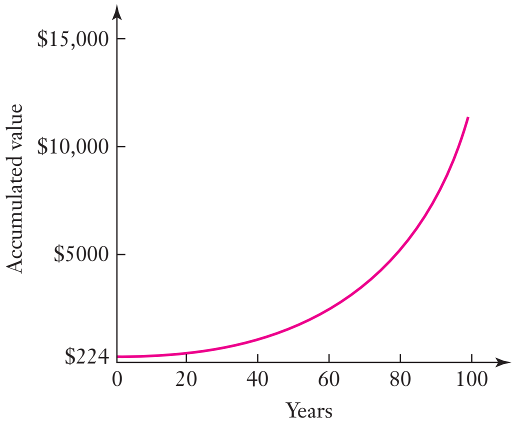


FIGURE 4.2 The value of the debt in the New College case during the first 100 years, at an interest rate of 4% per year. Note that the value rises much more rapidly in later years than in earlier years—a hallmark of exponential growth.

This rapid growth is a hallmark of what we generally call *exponential growth*. You can see how exponential growth gets its name by looking again at the general compound interest formula:

$$A = P \times (1 + \text{APR})^{Y}$$

Because the starting principal P and the interest rate APR have fixed values for any particular compound interest calculation, the growth of the accumulated value A depends only on Y (the number of times interest has been paid), which appears in the *exponent* of the calculation.

Exponential growth is one of the most important topics in mathematics, with applications that include population growth, resource depletion, and radioactivity. We will study exponential growth in much more detail in Chapter 8. In this chapter, we focus only on its applications in finance.

EXAMPLE 3 New College Debt at 2%

If the interest rate is 2%, calculate the amount due to New College using

a. simple interest **b.** compound interest

Solution

a. The following steps show the simple interest rate calculation for a starting principal $P = \$224$ and an annual interest rate of 2%:

1. The simple interest due each year is 2% of the starting principal: $2\% \times \$224 = 0.02 \times \$224 = \$4.48$
2. Over 535 years, the total interest due is: $535 \times \$4.48 = \$2396.80$
3. The total due after 535 years is the starting principal plus the interest: $\$224 + \$2396.80 = \$2620.80$

With simple interest, the payoff amount after 535 years is \$2620.80.

b. To find the amount due with compound interest, we set the annual interest rate to APR = 2% = 0.02 and the number of years to $Y = 535$. Then we use the formula for compound interest paid once a year:

$$
\begin{aligned}
A = P \times (1 + \text{APR})^Y &= \$224 \times (1 + 0.02)^{535} \\
&= \$224 \times (1.02)^{535} \\
&\approx \$224 \times 39{,}911 \\
&\approx \$8.94 \times 10^6
\end{aligned}
$$

The amount due with compound interest is about $8.94 million—far higher than the amount due with simple interest. ▶ **Now try Exercises 57–58.**

Effects of Interest Rate Changes

Notice the remarkable effects of small changes in the compound interest rate. In Example 3, we found that a 2% compound interest rate leads to a payoff amount of $8.94 million after 535 years. Earlier, we found that a 4% interest rate for the same 535 years leads to a payoff amount of $290 billion—which is more than 30,000 times as large as $8.94 million. Figure 4.3 contrasts the values of the New College debt during the first 100 years at interest rates of 2% and 4%. Note that the rate change doesn't make much difference for the first few years, but over time the higher rate becomes far more valuable.

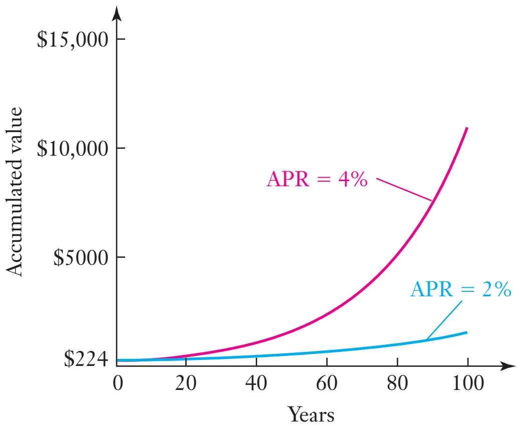


FIGURE 4.3 This figure contrasts the debt in the New College case during the first 100 years at interest rates of 2% and 4%.

Time Out to Think Suppose the interest rate for the New College debt were 3%. Without calculating, do you think the value after 535 years would be halfway between the values at 2% and 4% or closer to one or the other of these values? Now, check your guess by calculating the value at 3%. What happens at an interest rate of 6%? Briefly discuss why small changes in the interest rate can lead to large changes in the accumulated value.

EXAMPLE 4 Mattress Investments

Your grandfather put $100 under his mattress 50 years ago. If he had instead invested it in a bank account paying 3.5% interest compounded yearly (roughly the average U.S. rate of inflation during that period), how much would it be worth now?

Solution The starting principal is $P = \$100$. The annual percentage rate is $APR = 3.5\% = 0.035$. The number of years is $Y = 50$. So the accumulated balance is

$$\begin{aligned} A = P \times (1 + APR)^{Y} &= \$100 \times (1 + 0.035)^{50} \\ &= \$100 \times (1.035)^{50} \\ &= \$558.49 \end{aligned}$$

Invested at a rate of 3.5%, the \$100 would be worth over \$550 today. Unfortunately, the \$100 was put under a mattress, so it still has a face value of only \$100.

▶ **Now try Exercises 59–62.**

Compound Interest Paid More Than Once a Year

Suppose you could put \$1000 into an investment that pays compound interest at an annual percentage rate of APR = 8%. If the interest is paid all at once at the end of a year, you'll receive interest of

$$8\% \times \$1000 = 0.08 \times \$1000 = \$80$$

Therefore, your year-end balance will be \$1000 + \$80 = \$1080.

Now, assume instead that the investment pays interest *quarterly*, or four times a year (once every 3 months). The quarterly interest rate is one-fourth of the annual interest rate:

$$\text{quarterly interest rate} = \frac{APR}{4} = \frac{8\%}{4} = 2\% = 0.02$$

Table 4.3 shows how quarterly compounding affects the \$1000 starting principal during the first year.

TABLE 4.3 Quarterly Interest Payments (P = \$1000, APR = 8%)

After *N* Quarters	Interest Paid	New Balance
1st quarter (3 months)	2% × \$1000 = \$20	\$1000 + \$20 = \$1020
2nd quarter (6 months)	2% × \$1020 = \$20.40	\$1020 + \$20.40 = \$1040.40
3rd quarter (9 months)	2% × \$1040.40 = \$20.81	\$1040.40 + \$20.81 = \$1061.21
4th quarter (1 full year)	2% × \$1061.21 = \$21.22	\$1061.21 + \$21.22 = \$1082.43

Note that the year-end balance with quarterly compounding (\$1082.43) is *greater* than the year-end balance with interest paid all at once (\$1080). That is, when interest is compounded more than once a year, the balance increases by *more* than the APR in 1 year.

We can find the same results with the compound interest formula. Remember that the basic form of the compound interest formula is

$$A = P \times (1 + \text{interest rate})^{\text{number of compoundings}}$$

where A is the accumulated balance and P is the starting principal. In our current case, the starting principal is $P = \$1000$, the quarterly payments have an interest rate of $APR/4 = 0.02$, and in one year the interest is paid four times. Therefore, the accumulated balance at the end of one year is

$$A = P \times (1 + \text{interest rate})^{\text{number of compoundings}} = \$1000 \times (1 + 0.02)^{4} = \$1082.43$$

We see that if interest is paid quarterly, the interest rate at each payment is $APR/4$. Generalizing, if interest is paid n times per year, the interest rate at each payment is APR/n. The total number of times that interest is paid after Y years is nY. We therefore find the following formula for interest paid more than once each year.

Compound Interest Formula for Interest Paid *N* Times Per Year

$$A = P\left(1 + \frac{APR}{n}\right)^{(nY)}$$

where
A = accumulated balance after Y years
P = starting principal
APR = annual percentage rate (as a decimal)
n = number of compounding periods per year
Y = number of years

Note that Y is not necessarily an integer; for example, a calculation for six months would have $Y = 0.5$.

Time Out to Think Confirm that substituting $n = 1$ into the formula for interest paid n times per year gives you the formula for interest paid once a year. Explain why this should be true.

EXAMPLE 5 **Monthly Compounding at 3%**

You deposit \$5000 in a bank account that pays an APR of 3% and compounds interest monthly. How much money will you have after 5 years? Compare this amount to the amount you'd have if interest were paid only once each year.

Solution The starting principal is P = \$5000 and the interest rate is APR = 0.03. Monthly compounding means that interest is paid $n = 12$ times a year, and we are considering a period of $Y = 5$ years. We put these values into the compound interest formula to find the accumulated balance, A.

$$\begin{aligned} A = P \times \left(1 + \frac{APR}{n}\right)^{(nY)} &= \$5000 \times \left(1 + \frac{0.03}{12}\right)^{(12\times5)} \\ &= \$5000 \times (1.0025)^{60} \\ &= \$5808.08 \end{aligned}$$

For interest paid only once each year, we find the balance after 5 years by using the formula for compound interest paid once a year:

$$\begin{aligned} A = P \times (1 + APR)^{Y} &= \$5000 \times (1 + 0.03)^{5} \\ &= \$5000 \times (1.03)^{5} \\ &= \$5796.37 \end{aligned}$$

After 5 years, monthly compounding gives you a balance of \$5808.08 while annual compounding gives you a balance of \$5796.37. That is, monthly compounding earns \$5808.08 − \$5796.37 = \$11.71 more, even though the APR is the same in both cases.

▶ Now try Exercises 63–70.

Annual Percentage Yield (APY)

We've seen that in one year, money grows by *more* than the APR when interest is compounded more than once a year. For example, we found that with quarterly compounding and an 8% APR, a \$1000 principal increases to \$1082.43 in one year. This represents a relative increase of 8.24%:

$$\text{relative increase} = \frac{\text{absolute increase}}{\text{starting principal}} = \frac{\$82.43}{\$1000} = 0.08243 = 8.243\%$$

This relative increase over one year is called the **annual percentage yield (APY).** Note that it depends only on the annual interest rate (APR) and the number of compounding periods, not on the starting principal.

USING TECHNOLOGY

The Compound Interest Formula for Interest Paid More than Once a Year

Standard Calculators The procedure with interest paid more than once a year is essentially the same as that for the basic compound interest formula (see Using Technology, p. 230), except you enter APR/n instead of APR and nY instead of Y. Let's apply the procedure to Example 5, in which $P = \$5000$, APR $= 0.03$, $n = 12$, and $Y = 5$ years.

General Procedure	Our Example	Calculator Steps*	Output
$A = P \times \left(1 + \frac{APR}{n}\right)^{(nY)}$ 1. parentheses 2. exponent 3. multiply	$A = 5000 \times \left(1 + \frac{0.03}{12}\right)^{(12 \times 5)}$ 1. parentheses 2. exponent 3. multiply	Step 1 1 (+) 0.03 (÷) 12 (=)	1.0025
		Step 2 (∧) (( ) 12 (×) 5 ( )) (=)	1.1616...
		Step 3 (×) $5000 (=)	5808.08

*If your calculator does not have parentheses keys, then do the exponent $(nY = 12 \times 5)$ before you begin, keeping track of it on paper or in the calculator's memory.

Excel Use the built-in function FV just as for the basic compound interest formula (p. 230), *except*

- because *rate* is the interest rate *for each compounding period*, in this case use the monthly interest rate APR/n $= 0.03/12$.
- because *nper* is the total number of compounding periods, in this case use $nY = 12 \times 5$. (Note that Excel uses an asterisk * for multiplication.)

The following screen shot shows the direct entry of the FV function for our example.

```
fx  =FV(0.03/12,12*5,0,5000)
      FV(rate, nper, pmt, [pv], [type])
```

Again, it's best to show your work by referencing clearly labeled cells. In this case, we start with cells for APR, n, and Y, because these are the variables used in the compound interest formula in this book. These are then referenced to create the inputs for the FV function. You should create your own Excel worksheet to confirm that you get the result from Example 5 $(A = \$5808.08)$.

	A	B
1	APR	0.03
2	n (monthly compounding)	12
3	Y (number of years)	5
4	**rate** (for each compounding period = APR/n)	=B1/B2
5	**nper** (total number of compounding periods = nY)	=B2*B3
6	**pmt** (no monthly payment in this case)	0
7	**pv** (present value = starting principal P)	5000
8	**FV** (future value = accumulated balance A)	=FV(B4,B5,B6,B7)
9		