GLOBAL EDITION

Digital Control System Analysis and Design

FOURTH EDITION

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Digital Control System Analysis & Design

Fourth Edition

Global Edition

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(d) Find the conditions on $G(z)$ such that its dc gain is unbounded. Prove your result.

(e) For $G(z)$ as given in part (b), find the conditions of $G_p(s)$ such that the dc gain of $G(z)$ is unbounded. Prove your result.

4.3-5. Find the system response at the sampling instants to a unit-step input for the system of Fig. P4.3-5.

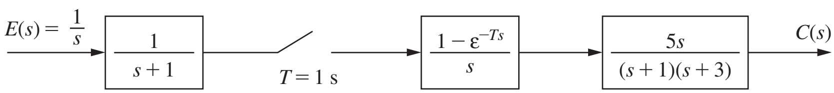


FIG. P4.3-5 System for Problem 4.3-5.

4.3-6. (a) Find the output $c(kT)$ for the system of Fig. P4.3-6, for $e(t)$ equal to a unit-step function.

(b) What is the effect on $c(kT)$ of the sampler and data hold in the upper path? Why?

(c) Sketch the unit-step response $c(t)$ of the system of Fig. P4.3-6. This sketch can be made without mathematically solving for $C(s)$.

(d) Repeat part (c) for the case that the sampler and data hold in the upper path is removed.

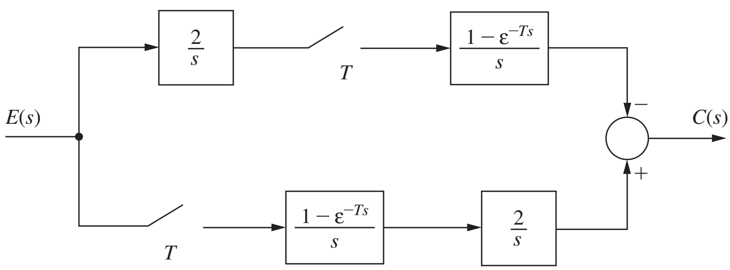
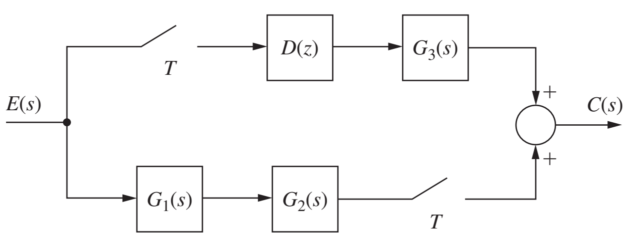


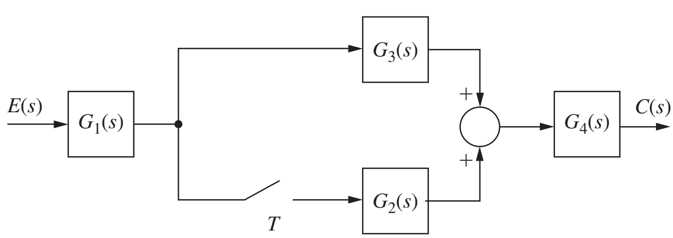
FIG. P4.3-6 System for Problem 4.3-6.

4.3-7. (a) Express each $C(s)$ and $C(z)$ as functions of the input for the systems of Fig. P4.3-7.

(b) List those transfer functions in Fig. P4.3-7 that contain the transfer function of a data hold.



(a)



(b)

FIGURE P4.3-7 Systems for Problem 4.3-7.

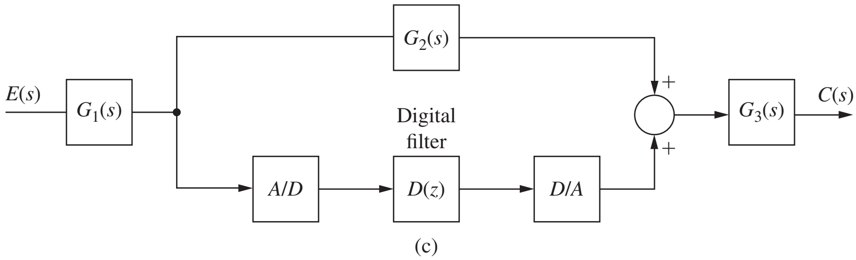


FIGURE P4.3-7 (*continued*)

4.3-8. Shown in Fig. P4.3-8 is the block diagram of one joint of a robot arm. This system is described in Problem 1.5-4. The signal $M(s)$ is the sampler input, $E_a(s)$ is the servomotor input voltage, $\Theta_m(s)$ is the motor shaft angle, and the output $\Theta_a(s)$ is the angle of the arm.

(a) Suppose that the sampling-and-data-reconstruction process is implemented with an analog-to-digital converter (A/D) and a digital-to-analog converter. Redraw Fig. P4.3-8 showing the A/D and the D/A.

(b) Suppose that the units of $e_a(t)$ are volts, and of $\Theta_m(t)$ are rpm. The servomotor is rated at 24 V (the voltage $e_a(t)$ should be less than or equal to 24 V in magnitude). Commercially available D/As are usually rated with output voltage ranges of ± 5 V, ± 10 V, 0 to 5 V, 0 to 10 V, or 0 to 20 V. If the gain of the power amplifier is 2.4, what should be the rated voltage of the D/A? Why?

(c) Derive the analog transfer function $\Theta_a(s)/E_a(s)$.

(d) With $K = 2.4$ and $T = 0.1$ s, derive the pulse transfer function $\Theta_a(z)/M(z)$.

(e) Derive the steady-state output for $m(t)$ constant. Justify this value from the motor characteristics.

(f) Verify the results of part (d) by computer.

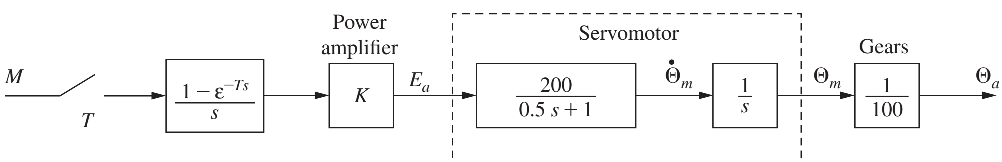


FIG. P4.3-8 Model of robot arm joint.

4.3-9 Fig. P4.3-9 illustrates a thermal stress chamber. This system is described in Problem 1.6-1. The system output $c(t)$ is the chamber temperature in degrees Celsius, and the control-voltage input $m(t)$ operates a valve on a steam line. The sensor is based on a thermistor, which is a temperature-sensitive resistor. The disturbance input $d(t)$ models the opening of the door into the chamber. With the door closed, $d(t) = 0$; if the door is opened at $t = t_0$, $d(t) = u(t - t_0)$, a unit-step function.

(a) Suppose that the sampling-and-data-reconstruction process is implemented with an analog-to-digital converter (A/D) and a digital-to-analog converter (D/A). Redraw Fig. P4.3-9 showing the A/D and the D/A.

(b) Derive the transfer function $C(z)/E(z)$.

(c) A constant voltage of $e(t) = 10$ V is applied for a long period. Find the steady-state temperature of the chamber, with the door closed. Note that this problem can be solved without knowing the sample period T.

(d) Find the steady-state effect on the chamber temperature of leaving the door open.
(e) Find the expression for $C(s)$ as a function of the Laplace-transform variable s, the control input, and the disturbance input. The z-transform variable z cannot appear in this expression.

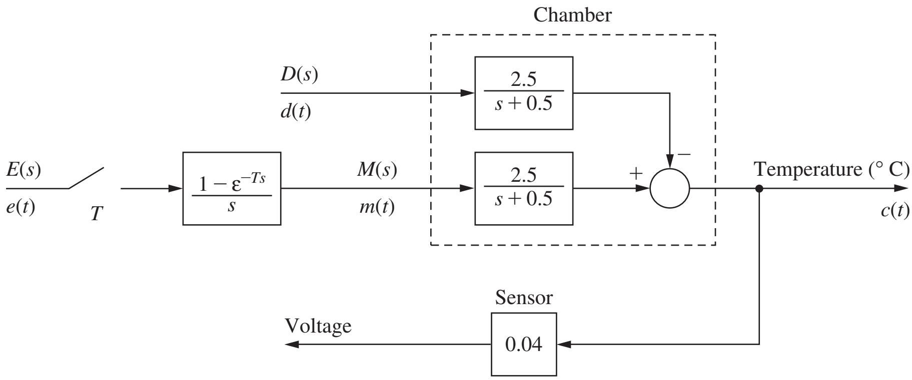


FIGURE P4.3-9 Block diagram for a thermal test chamber.

4.3-10. Given in Fig. P4.3-10 is the block diagram of a rigid-body satellite. The control signal is the voltage $e(t)$. The zero-order hold output $m(t)$ is converted into a torque $\tau(t)$ by an amplifier and the thrusters (see Section 1.4). The system output is the attitude angle $\theta(t)$ of the satellite.
(a) Find the transfer function $\Theta(z)/E(z)$.
(b) Use the results of part (a) to find the system's unit-step response, that is, the response with $e(t) = u(t)$.
(c) Sketch the zero-order-hold output $m(t)$ in (b).
(d) Use $m(t)$ in part (c) to find $c(t) = \mathscr{L}^{-1}[KM(s)/Js^2]$.
(e) In part (d), evaluate $c(kT)$. This response should equal that found in part (b).

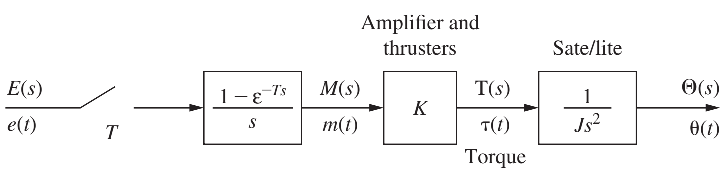


FIG. P4.3-10 Block diagram for a satellite.

4.3-11. The antenna positioning system described in Section 1.5 and Problem 1.5-1 is depicted in Fig. P4.3-11. In this problem we consider the yaw angle control system, where $\theta(t)$ is the yaw angle. The angle sensor (a digital shaft encoder and the data hold) yields $v_o(kT) = [0.4\,\theta(kT)]$, where the units of $v_o(t)$ are volts and $\theta(t)$ are degrees. The sample period is $T = 0.05$ s.
(a) Find the transfer function $\Theta(z)/E(z)$.
(b) The yaw angle is initially zero. The input voltage $e(t)$ is set equal to 10 V at $t = 0$, and is zero at each sample period thereafter. Find the steady-state value of the yaw angle.
(c) Note that in part (b), the coefficients in the partial-fraction expansion add to zero. Why does this occur?
(d) The input voltage $e(t)$ is set to a constant value. Without solving mathematically, give a description of the system response.

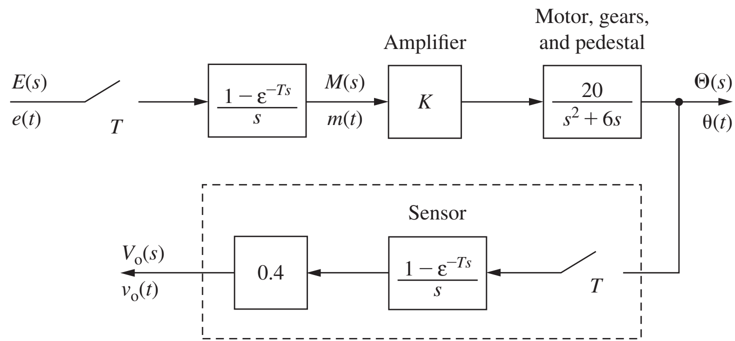


FIGURE P4.3-11 Block diagram for an antenna control system.

(e) Suppose in part (d) that you are observing the antenna. Describe what you would see.

4.4-1. Example 4.3 calculates the step response of the system in Fig. 4-2. Example 4.4 calculates the step response of the same system preceded by a digital filter with the transfer function $D(z) = (2 - z^{-1})$. This system is shown in Fig. P4.4-1.

(a) Solve for the output of the digital filter $m(kT)$.

(b) Let the response in Example 4.3 be denoted as $c_1(kT)$. Use the results in part (a) to express the output $c(kT)$ in Fig. P4.4-1 as a function of $c_1(kT)$.

(c) Use the response $c_1(kT)$ calculated in Example 4.3 and the results in part (b) to find the output $c(kT)$ in Fig. P4.4-1. This result should be the same as in Example 4.4.

(d) Use the response $C_1(z)$ calculated in Example 4.3 and the result in part (b) to find the output $C(z)$ in Fig. P4.4-l. This result should be the same as in Example 4.4.

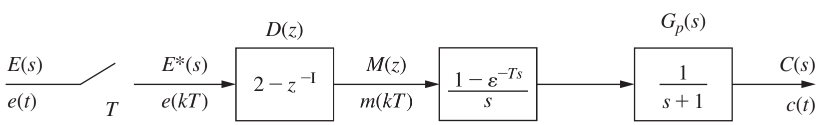


FIG. P4.4-1 System for Problem 4.4-1.

4.4-2. Consider the *hardware* depicted in Fig. P4.4-2. The transfer function of the digital controller implemented in the computer is given by

$$D(z) = \frac{4.5(z - 0.90)}{z - 0.85}$$

The input voltage rating of the analog-to-digital converter is ± 10 V and the output voltage rating of the digital-to-analog converter is also ± 10 V.

(a) Calculate the dc gain of the controller.

(b) State exactly how you would verify, using the hardware, the value calculated in part (a). Give the equipment required, the required settings on the equipment, and the expected measurements.

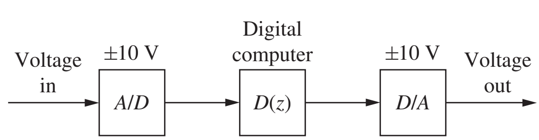


FIGURE P4.4-2 Hardware configuration for Problem 4.4-2.

4.4-3. For the system of Fig. P4.4-3, the filter solves the difference equation

$$m(k) = 0.8m(k - 1) + 0.1e(k)$$

The sampling rate is 1 Hz and the plant transfer function is given by

$$G_p(s) = \frac{1}{s + 0.2}$$

(a) Find the system transfer function $C(z)/E(z)$.
(b) Find the system dc gain from the results of part (a).
(c) Verify the results of part (b) by finding the dc gain of the filter using $D(z)$ and that of the plant using $G_p(s)$.
(d) Use the results of part (b) to find the steady-state value of the unit-step response.
(e) Verify the results of part (d) by calculating $c(kT)$ for a unit-step input.
(f) Note that in part (e), the coefficients in the partial-fraction expansion add to zero. Why does this occur?

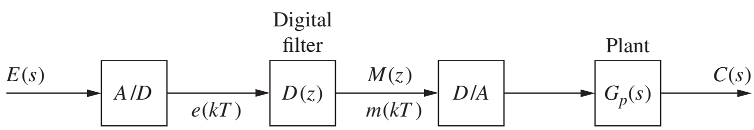


FIGURE P4.4-3 System for Problems 4.4-3 and 4.4-4.

4.4-4. Repeat Problem 4.4-3 for the case that the filter solves the difference equation

$$m(k + 1) = 0.5e(k + 1) - (0.5)(0.8)e(k) + 0.485m(k),$$

the sampling rate is 10 Hz, and the plant transfer function is given by

$$G_p(s) = \frac{5}{(s + 1)(s + 2)}$$

4.4-5. Consider the system of Fig. P4.4-5. The filter transfer function is $D(z)$.
(a) Express $C(z)$ as a function of E.
(b) A discrete state model of this system does not exist. Why?
(c) What assumptions concerning $e(t)$ must be made in order to derive an approximate discrete state model?

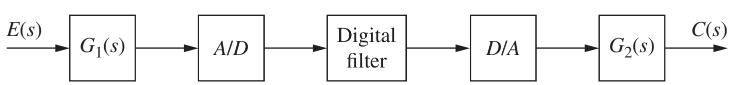


FIGURE P4.4-5 System for Problem 4.4-5.

4.4-6. Consider again the system of Fig. P4.4-5. Add a sampler for $E(s)$ at the input. Given

$$G_1(s) = \frac{1}{s + 10} \qquad D(z) = \frac{z - 0.5}{z - 1} \qquad G_2(s) = \frac{s}{s^2 + 9s + 23}$$

find $c(kT)$ for a unit-step input with $T = 0.5$ s. Is this a good choice for the sampling interval? If not, what sampling rate would you recommend. Find $c(kT)$ at your recommended sampling rate and plot both step responses using MATLAB.

4.5-1. Find the modified z-transform of the following functions:

(a) $E(s) = \dfrac{20}{(s+2)(s+5)}$ (b) $E(s) = \dfrac{5}{s(s+1)}$

(c) $E(s) = \dfrac{s+2}{s(s+1)}$ (d) $E(s) = \dfrac{s+2}{s^2(s+1)}$

(e) $E(s) = \dfrac{s^2+5s+6}{s(s+4)(s+5)}$ (f) $E(s) = \dfrac{2}{s^2+2s+5}$

4.5-2. Find the z-transform of the following functions. The results of Problem 4.5-1 may be useful.

(a) $E(s) = \dfrac{20\varepsilon^{-0.3Ts}}{(s+2)(s+5)}$ (b) $E(s) = \dfrac{5\varepsilon^{-0.6Ts}}{s(s+1)}$

(c) $E(s) = \dfrac{s+2\varepsilon^{-1.1Ts}}{s(s+1)}$ (d) $E(s) = \dfrac{s+2\varepsilon^{-0.2Ts}}{s^2(s+1)}$

(e) $E(s) = \dfrac{(s^2+5s+6)\varepsilon^{-0.3Ts}}{s(s+4)(s+5)}$ (f) $E(s) = \dfrac{2\varepsilon^{-0.75Ts}}{s^2+2s+5}$

4.5-3. Find the modified z-transform of the following functions:

(a) $E(s) = \dfrac{6}{(s+1)(s+2)(s+3)}$ (b) $E(s) = \dfrac{4}{s(s+2)^2}$

(c) $E(s) = \dfrac{s^2+2s+2}{s(s+2)^2}$ (d) $E(s) = \dfrac{(s+2)^2}{s^2(s+1)^2}$

(e) $E(s) = \dfrac{s^2+2s+3}{s(s+2)^2(s+4)}$ (f) $E(s) = \dfrac{2s+7}{(s^2+2s+5)(s+3)}$

4.5-4. Find the z-transform of the following functions. The results of Problem 4.5-3 may be useful.

(a) $E(s) = \dfrac{6\varepsilon^{-0.3Ts}}{(s+1)(s+2)(s+3)}$ (b) $E(s) = \dfrac{4\varepsilon^{-0.6Ts}}{s(s+2)^2}$

(c) $E(s) = \dfrac{s^2+2s+2\varepsilon^{-1.1Ts}}{s(s+2)^2}$ (d) $E(s) = \dfrac{(s+2)^2\varepsilon^{-0.2Ts}}{s^2(s+1)^2}$

(e) $E(s) = \dfrac{s^2\varepsilon^{-0.3Ts}+2s+3}{s(s+2)^2(s+4)}$ (f) $E(s) = \dfrac{(2s+7)\varepsilon^{-0.75Ts}}{(s^2+2s+5)(s+3)}$

4.6-1. Generally, a temperature control system is modeled more accurately if an ideal time delay is added to the plant. Suppose that in the thermal test chamber of Problem 4.3-9, the plant transfer function is given by

$$G_p(s) = \frac{C(s)}{E(s)} = \frac{2\varepsilon^{-2s}}{s+0.5}$$

Hence the plant has a 2-s time delay before its response to an input. For this problem, let the sample period $T = 0.6$ s.

(a) Find the unit step response for the system of Fig. P4.3-9; that is, find $c(kT)$ with $e(t) = u(t)$ and $d(t) = 0$, and with no delay.

(b) Repeat part (a), with the 2-s time delay included in $G_p(s)$, as given above.