Elementary Surveying

An Introduction to Geomatics

FOURTEENTH EDITION

Charles D. Ghilani • Paul R. Wolf

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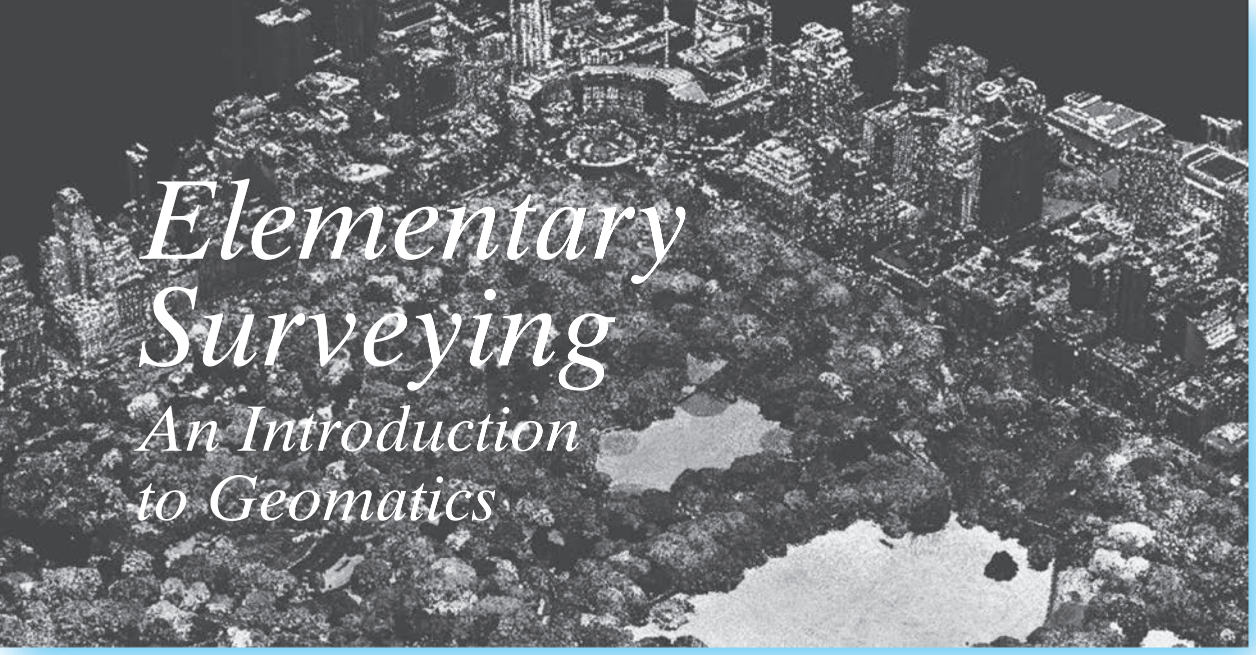

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An Introduction to Geomatics

Fourteenth Edition

Global Edition

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10.21 The following data apply to a closed link traverse [like that of Figure 9.1(b)]. Compute preliminary azimuths, adjust them, and calculate departures and latitudes, misclosures in departure and latitude, and traverse relative precision. Balance the departures and latitudes using the compass rule, and calculate coordinates of points *B, C*, and *D*. Compute the final lengths and azimuths of lines *AB, BC, CD*, and *DE*.

Station	Measured Angle (to the Right)	Adjusted Azimuth	Measured Length (ft)	*X* (ft)	*Y* (ft)
$AzMk_1$					
		342°09′28″			
A	258°12′18″			2,521,005.86	379,490.84
			200.55		
B	215°02′53″				
			253.84		
C	128°19′11″				
			205.89		
D	237°34′05″			2,521,575.16	379,714.76
		101°18′31″			
$AzMk_2$					

10.22 Similar to Problem 10.21, except use the following data:

Station	Measured Angle (to the Right)	Adjusted Azimuth	Measured Length (m)	*X* (m)	*Y* (m)
$AzMk_1$					
		250°57′23″			
A	253°03′38″			194,325.090	25,353.988
			224.111		
B	91°32′06″				
			116.738		
C	242°25′54″				
			231.566		
D	111°12′02″				
			97.217		
E	295°31′13″			193,819.150	25,514.391
		344°42′26″			
$AzMk_2$					

The azimuths (from north of a polygon traverse are $AB = 38°17'02''$, $BC = 121°26'30''$, $CD = 224°56'59''$, and $DA = 308°26'56''$. If one observed distance contains a mistake, which course is most likely responsible for the closure conditions given in Problems 10.23 and 10.24? Is the course too long or too short?

10.23* Algebraic sum of departures = 5.12 ft latitudes = −3.13 ft.

10.24 Algebraic sum of departures = −3.133 m latitudes = +2.487 m.

10.25 Determine the lengths and bearings of the sides of a lot whose corners have the following *X* and *Y* coordinates (in feet): *A* (5000.00, 5000.00); *B* (5289.67, 5436.12); *C* (4884.96, 5354.54); *D* (4756.66, 5068.37).

10.26 Compute the lengths and azimuths of the sides of a closed-polygon traverse, whose corners have the following *X* and *Y* coordinates (in meters): *A* (1200.000, 3000.000); *B* (1347.790, 1408.926); *C* (2093.028, 1628.853); *D* (2190.303, 1954.624).

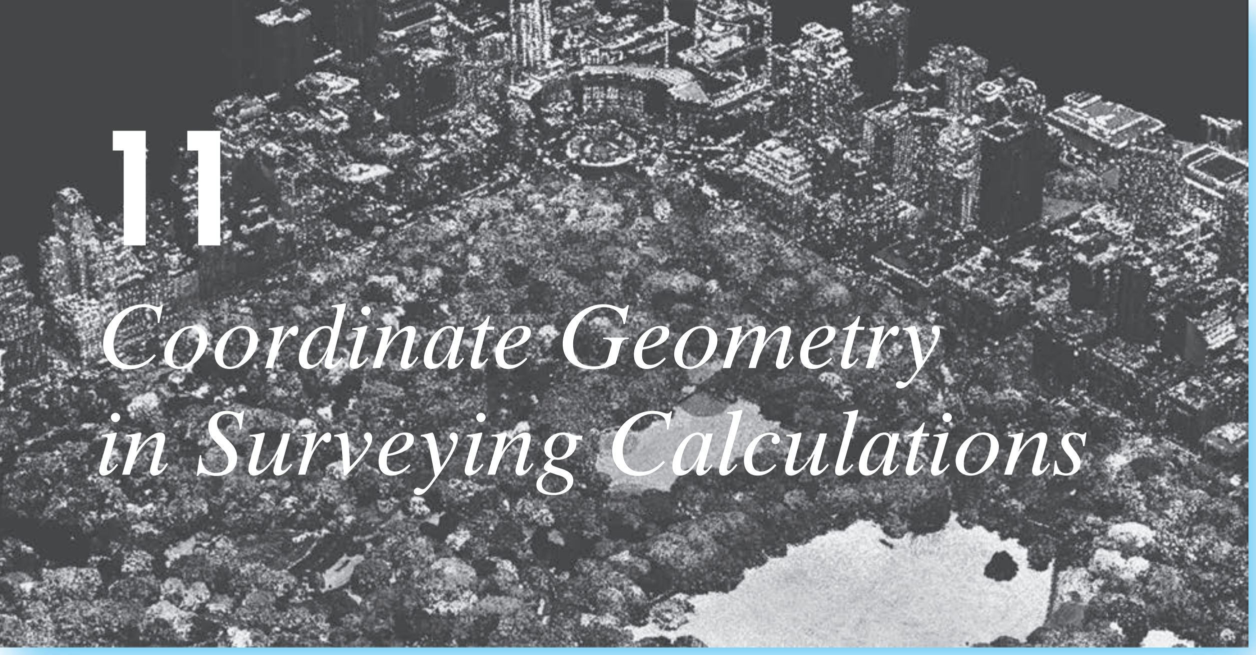
10.27 In searching for a record of the length and true bearing of a certain boundary line which is straight between *A* and *B*, the following notes of an old random traverse were found (survey by compass and Gunter's chain, declination 4°45′W). Compute the true bearing and length (in feet) of *BA*.

Course	*A*-1	1-2	2-3	3-*B*
Magnetic bearing	Due North	N20°00′E	Due East	S46°30′E
Distance (ch)	11.90	35.80	24.14	12.72

10.28 Describe how a blunder may be located in a traverse.

BIBLIOGRAPHY

Ghilani, C. D. 2010. *Adjustment Computations: Spatial Data Analysis*, 5th Ed. New York, NY: Wiley.



11

Coordinate Geometry in Surveying Calculations

11.1 INTRODUCTION

Except for extensive geodetic control surveys, almost all other surveys are referenced to plane rectangular coordinate systems. State plane coordinates (see Chapter 20) are most frequently employed, although local arbitrary systems can be used. Advantages of referencing points in a rectangular coordinate system are as follows: (1) the relative positions of points are uniquely defined, (2) they can be conveniently plotted, (3) if lost in the field, they can readily be recovered from other available points referenced to the same system, and (4) computations are greatly facilitated.

Computations involving coordinates are performed in a variety of surveying problems. Two situations were introduced in Chapter 10, where it was shown that the length and direction (azimuth or bearing) of a line can be calculated from the coordinates of its end points. Area computation using coordinates is discussed in Chapter 12. Additional problems that are conveniently solved using coordinates are determining the point of intersection of (a) two lines, (b) a line and a circle, and (c) two circles. The solutions for these and other coordinate geometry problems are discussed in this chapter. It will be shown that the method employed to determine the intersection point of a line and a circle reduces to finding the intersection of a line of known azimuth and another line of known length. Also, the problem of finding the intersection of two circles consists of determining the intersection point of two lines having known lengths. These types of problems are regularly encountered in the horizontal alignment surveys where it is necessary to compute intersections of tangents and circular curves, and in boundary and subdivision work where parcels of land are often defined by straight lines and circular arcs.

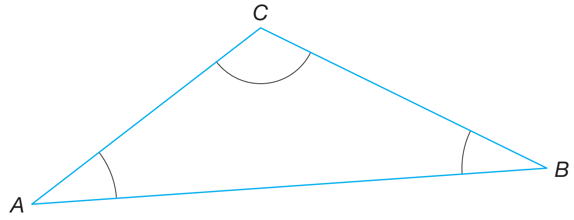


Figure 11.1
An oblique triangle.

The three types of intersection problems noted above are conveniently solved by forming a triangle between two stations of known position from which the observations are made, and then solving for the parts of this triangle. Two important functions used in solving oblique triangles are (1) the law of sines, and (2) the law of cosines. The law of sines relates the lengths of the sides of a triangle to the sines of the opposite angles. For Figure 11.1, this law is

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C} \tag{11.1}$$

where AB, BC, and AC are the lengths of the three sides of the triangle ABC, and A, B, and C are the angles. The law of cosines relates two sides and the included angle of a triangle to the length of the side opposite the angle. In Figure 11.1, the following three equations can be written that express the law of cosines:

$$\begin{aligned} BC^2 &= AC^2 + AB^2 - 2(AC)(AB)\cos A \\ AC^2 &= BA^2 + BC^2 - 2(BA)(BC)\cos B \\ AB^2 &= CB^2 + CA^2 - 2(CB)(CA)\cos C \end{aligned} \tag{11.2}$$

In some coordinate geometry solutions, the use of the quadratic formula can be used. Examples where this equation simplifies the solution are discussed in Sections 24.16.1 and 25.10. This formula, which gives the solution for x in any quadratic equation of form $ax^2 + bx + c = 0$, is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{11.3}$$

In the remaining sections of this chapter, procedures using triangles and Equations (11.1) through (11.3) are presented for solving each type of standard coordinate geometry problem.

11.2 COORDINATE FORMS OF EQUATIONS FOR LINES AND CIRCLES

In Figure 11.2, straight line AB is referenced in a plane rectangular coordinate system. Coordinates of end points A and B are X_A, Y_A, X_B, and Y_B. Length AB and azimuth Az_{AB} of this line in terms of these coordinates are

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \tag{11.4}$$

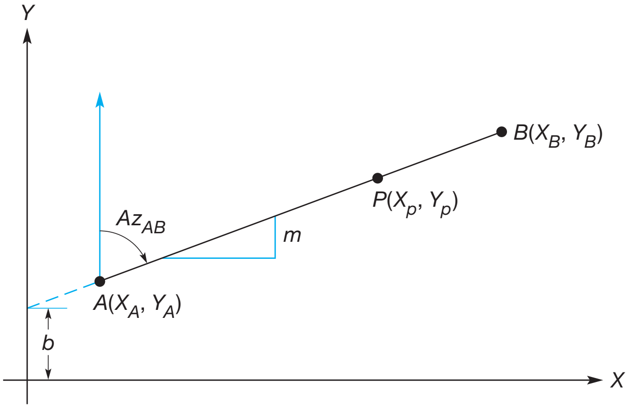


Figure 11.2 Geometry of a straight line in a plane coordinate system.

$$Az_{AB} = \tan^{-1}\left(\frac{\Delta X}{\Delta Y}\right) + C \tag{11.5a}$$

where ΔX is $X_B - X_A$, ΔY is $\Delta Y_B - \Delta Y_A$, C is 0° if both ΔX and ΔY are greater than zero, C is 180° if ΔY is less than zero, and C is 360° if ΔX is less than zero, and ΔY is greater than zero. Another frequently used equation for determining the azimuth of a course in software is known as the atan2 function, which is computed as

$$Az_{AB} = \text{atan2}(\Delta Y, \Delta X) + D = 2\tan^{-1}\left(\frac{\sqrt{\Delta X^2 + \Delta Y^2} - \Delta Y}{\Delta X}\right) + D \tag{11.5b}$$

where D is the 0° if the results of the atan2 function are positive and 360° if the results of the function are negative. The general mathematical expression for a straight line is

$$Y_P = mX_P + b \tag{11.6}$$

where Y_P is the Y coordinate of any point P on the line whose X coordinate is X_P, m the slope of the line, and b the y-intercept of the line. Slope m can be expressed as

$$m = \frac{Y_B - Y_A}{X_B - X_A} = \cot(Az_{AB}) \tag{11.7}$$

From Equations (11.5a) and (11.7), it can be shown that

$$Az_{AB} = \tan^{-1}\left(\frac{1}{m}\right) + C \tag{11.8}$$

The mathematical expression for a circle in rectangular coordinates can be written as

$$R^2 = (X_P - X_O)^2 + (Y_P - Y_O)^2 \tag{11.9}$$

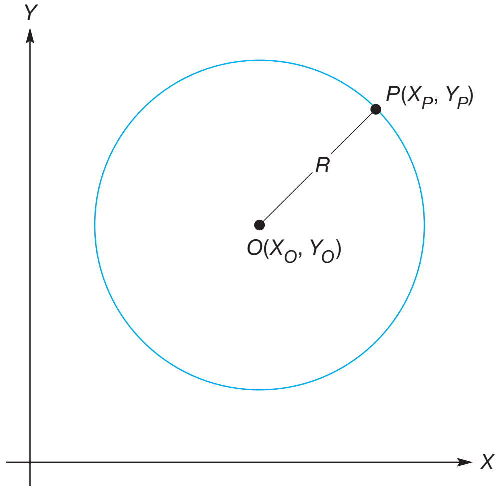


Figure 11.3 Geometry of a circle in a plane coordinate system.

In Equation (11.9), and with reference to Figure 11.3, R is the radius of the circle, X_O and Y_O are the coordinates of the radius point O, and X_P and Y_P the coordinates of any point P on the circle. The general form of the circle equation is

$$X_P^2 + Y_P^2 - 2X_OX_P - 2Y_OY_P + f = 0 \tag{11.10}$$

where the radius of the circle is given as $R = \sqrt{X_O^2 + Y_O^2 - f}$. [*Note:* Although Equations (11.9) and (11.10) are not used in solving problems in this chapter, they are applied in later chapters.]

11.3 PERPENDICULAR DISTANCE FROM A POINT TO A LINE

A common problem encountered in boundary surveying is determining the perpendicular distance of a point from a line. This procedure can be used to check the alignment of survey markers on a block and is also useful in subdivision design. Assume in Figure 11.4 that points A and B are on the line defined by two block corners whose coordinates are known. Also assume that the

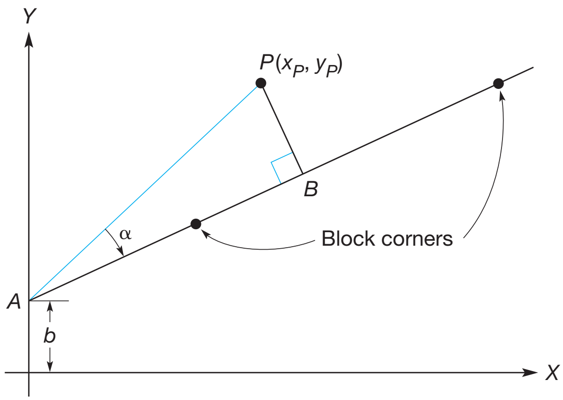


Figure 11.4 Perpendicular distance of a point from a line.