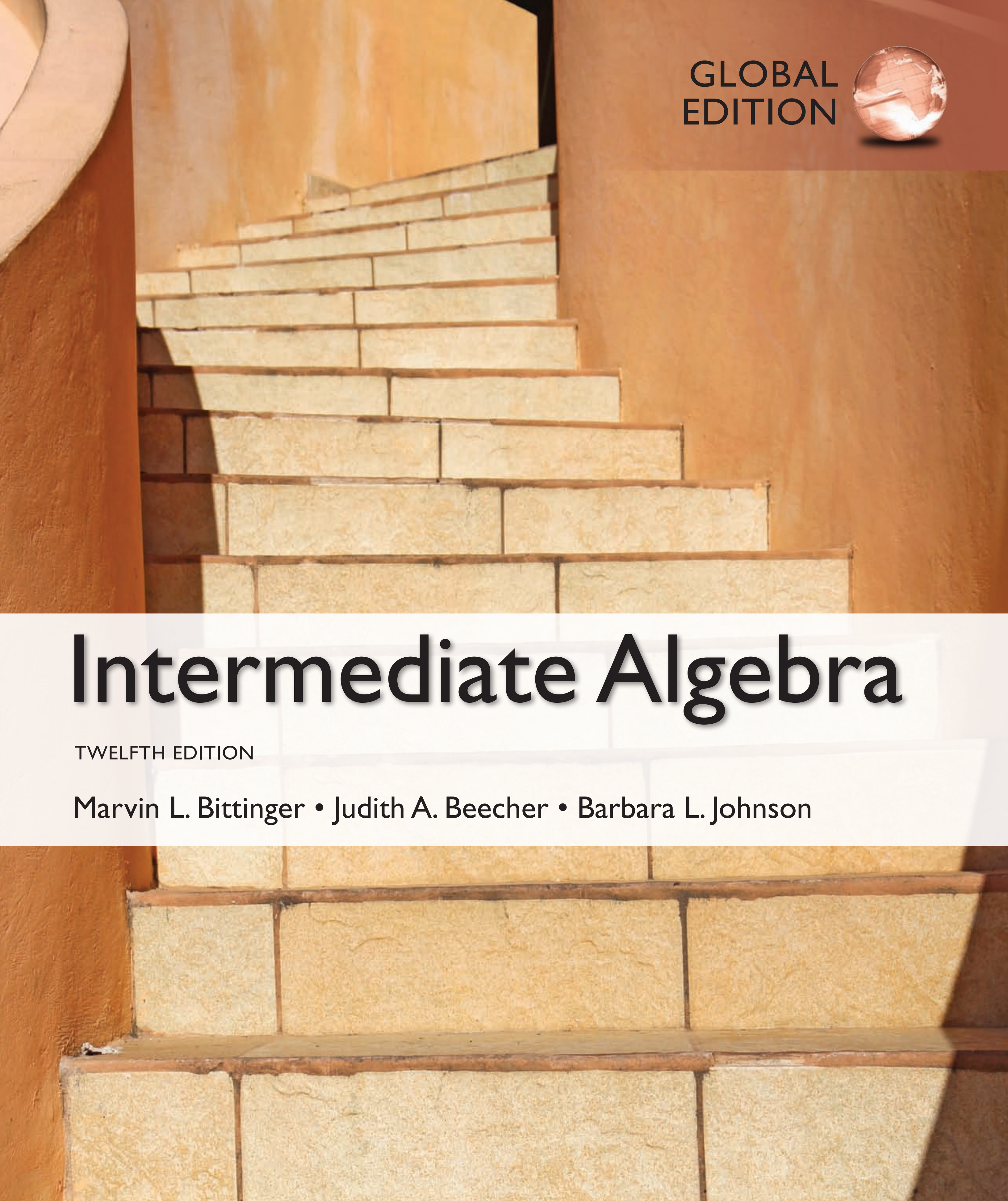

GLOBAL EDITION

Intermediate Algebra

TWELFTH EDITION

Marvin L. Bittinger • Judith A. Beecher • Barbara L. Johnson

ALWAYS LEARNING

PEARSON

Intermediate Algebra

TWELFTH EDITION

GLOBAL EDITION

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PEARSON

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3. Solve by the elimination method:

$$\begin{aligned} 2y + 3x &= 12, & \textbf{(1)} \\ -4y + 5x &= -2. & \textbf{(2)} \end{aligned}$$

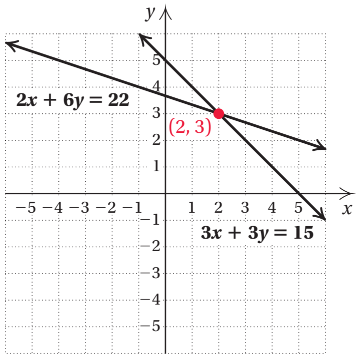
Multiply by 2 on both sides of equation (1) and add:

$$\begin{aligned} 4y + 6x &= 24 \\ \underline{-4y + 5x} &\underline{= -2} \\ 0 + \underline{\quad} &= \underline{\quad} \\ 11x &= 22 \\ x &= \underline{\quad}. \end{aligned}$$

Substitute $\underline{\quad}$ for x in equation (1) and solve for y:

$$\begin{aligned} 2y + 3x &= 12 \\ 2y + 3(\underline{\quad}) &= 12 \\ 2y + 6 &= 12 \\ 2y &= \underline{\quad} \\ y &= \underline{\quad}. \end{aligned}$$

The ordered pair checks in both equations, so the solution is $(\underline{\quad}, \underline{\quad})$.



In order to eliminate a variable, we sometimes use the multiplication principle to multiply one or both of the equations by a particular number before adding.

EXAMPLE 2 Solve this system:

$$\begin{aligned} 3x + 3y &= 15, & \textbf{(1)} \\ 2x + 6y &= 22. & \textbf{(2)} \end{aligned}$$

If we add directly, we will not eliminate a variable. However, note that if the $3y$ in equation (1) were $-6y$, we could eliminate y. Thus we multiply by -2 on both sides of equation (1) and add:

$$\begin{aligned} -6x - 6y &= -30 && \text{Multiplying by } -2 \text{ on both sides of equation (1)} \\ 2x + 6y &= 22 && \text{Equation (2)} \\ -4x + 0 &= -8 && \text{Adding} \\ -4x &= -8 \\ x &= 2. && \text{Solving for } x \end{aligned}$$

Then

$$\begin{aligned} 2 \cdot 2 + 6y &= 22 && \text{Substituting 2 for } x \text{ in equation (2)} \\ 4 + 6y &= 22 \\ 6y &= 18 && \text{Solving for } y \\ y &= 3. \end{aligned}$$

We obtain $(2, 3)$, or $x = 2$, $y = 3$.

Check:

$$\begin{array}{r|l} 3x + 3y & = 15 \\ \hline 3(2) + 3(3) & ?\ 15 \\ 6 + 9 & \\ 15 & \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} 2x + 6y & = 22 \\ \hline 2(2) + 6(3) & ?\ 22 \\ 4 + 18 & \\ 22 & \quad \text{TRUE} \end{array}$$

Since $(2, 3)$ checks, it is the solution. We can also see this in the graph at left.

Do Exercise 3.

Sometimes we must multiply twice in order to make two terms opposites.

EXAMPLE 3 Solve this system:

$$\begin{aligned} 2x + 3y &= 17, & \textbf{(1)} \\ 5x + 7y &= 29. & \textbf{(2)} \end{aligned}$$

We must first multiply in order to make one pair of terms with the same variable opposites. We decide to do this with the x-terms in each equation. We multiply equation (1) by 5 and equation (2) by -2. Then we get $10x$ and $-10x$, which are opposites.

$$\begin{aligned} &\textit{From equation (1):} & 10x + 15y &= 85 && \text{Multiplying by 5} \\ &\textit{From equation (2):} & -10x - 14y &= -58 && \text{Multiplying by } -2 \\ && 0 + y &= 27 && \text{Adding} \\ && y &= 27 && \text{Solving for } y \end{aligned}$$

Answer

3. (2, 3)

Guided Solution:

3. $11x$, 22, 2, 2, 2, 6, 3, 2, 3

Then

$$\begin{aligned} 2x + 3 \cdot 27 &= 17 && \text{Substituting 27 for } y \text{ in equation (1)} \\ 2x + 81 &= 17 \\ 2x &= -64 && \text{Solving for } x \\ x &= -32. \end{aligned}$$

We check the ordered pair $(-32, 27)$.

Check:

$$\begin{array}{c|l} 2x + 3y & = 17 \\ \hline 2(-32) + 3(27) & ?\ 17 \\ -64 + 81 & \\ 17 & \quad \text{TRUE} \end{array} \qquad \begin{array}{c|l} 5x + 7y & = 29 \\ \hline 5(-32) + 7(27) & ?\ 29 \\ -160 + 189 & \\ 29 & \quad \text{TRUE} \end{array}$$

We obtain $(-32, 27)$, or $x = -32$, $y = 27$, as the solution.

Do Exercises 4 and 5. ▶

When solving a system of equations using the elimination method, it helps to first write the equations in the form $Ax + By = C$. When decimals or fractions occur, it also helps to *clear* before solving.

EXAMPLE 4 Solve this system:

$$0.2x + 0.3y = 1.7,$$
$$\tfrac{1}{7}x + \tfrac{1}{5}y = \tfrac{29}{35}.$$

We have

$$0.2x + 0.3y = 1.7, \longrightarrow \text{Multiplying by 10 to clear decimals} \longrightarrow 2x + 3y = 17,$$
$$\tfrac{1}{7}x + \tfrac{1}{5}y = \tfrac{29}{35} \longrightarrow \text{Multiplying by 35 to clear fractions} \longrightarrow 5x + 7y = 29.$$

We multiplied by 10 to clear the decimals. Multiplication by 35, the least common denominator, clears the fractions. The problem is now identical to Example 3. The solution is $(-32, 27)$, or $x = -32$, $y = 27$.

Do Exercises 6 and 7. ▶

To use the elimination method to solve systems of two equations:

1. Write both equations in the form $Ax + By = C$.
2. Clear any decimals or fractions.
3. Choose a variable to eliminate.
4. Make the chosen variable's terms opposites by multiplying one or both equations by appropriate numbers if necessary.
5. Eliminate a variable by adding the respective sides of the equations and then solve for the remaining variable.
6. Substitute in either of the original equations to find the value of the other variable.

Solve by the elimination method.

4. $4x + 5y = -8$,
$7x + 9y = 11$

5. $4x - 5y = 38$,
$7x - 8y = -22$

6. Clear the decimals. Then solve.

$$0.02x + 0.03y = 0.01,$$
$$0.3x - 0.1y = 0.7$$

(*Hint*: Multiply the first equation by 100 and the second one by 10.)

7. Clear the fractions. Then solve.

$$\frac{3}{5}x + \frac{2}{3}y = \frac{1}{3},$$
$$\frac{3}{4}x - \frac{1}{3}y = \frac{1}{4}$$

Answers

4. $(-127, 100)$ **5.** $(-138, -118)$
6. $2x + 3y = 1$,
$3x - y = 7$; $(2, -1)$
7. $9x + 10y = 5$,
$9x - 4y = 3$; $\left(\frac{25}{63}, \frac{1}{7}\right)$

Some systems have no solution. How do we recognize such systems if we are solving using elimination?

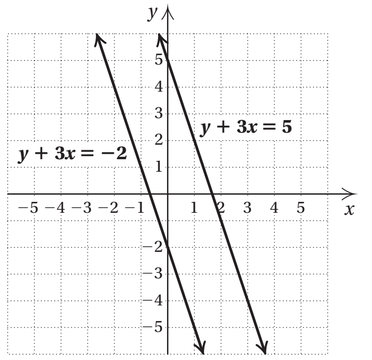
EXAMPLE 5 Solve this system:

$$y + 3x = 5, \quad \textbf{(1)}$$
$$y + 3x = -2. \quad \textbf{(2)}$$

If we find the slope–intercept equations for this system, we get

$$y = -3x + 5,$$
$$y = -3x - 2.$$

The graphs are parallel lines.
The system has no solution.



Let's attempt to solve the system by the elimination method:

$$\begin{aligned} y + 3x &= 5 &&\text{Equation (1)}\\ -y - 3x &= 2 &&\text{Multiplying equation (2) by } -1\\ \hline 0 &= 7. &&\text{Adding, we obtain a false equation.} \end{aligned}$$

The x-terms and the y-terms are eliminated and we have a *false* equation. If we obtain a false equation, such as $0 = 7$, when solving algebraically, we know that the system has **no solution**. The system is inconsistent, and the equations are independent.

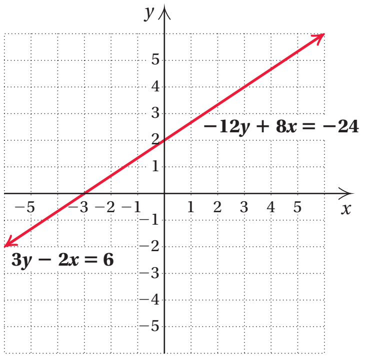
Do Exercise 8.

Some systems have infinitely many solutions. How can we recognize such a situation when we are solving systems using an algebraic method?

EXAMPLE 6 Solve this system:

$$3y - 2x = 6, \quad \textbf{(1)}$$
$$-12y + 8x = -24. \quad \textbf{(2)}$$

The graphs are the same line. The system has an infinite number of solutions.



8. Solve by the elimination method: GS

$$y + 2x = 3,$$
$$y + 2x = -1.$$

Multiply the second equation by -1 and add:

$$\begin{aligned} y + 2x &= 3\\ -y - 2x &= 1\\ \hline 0 &= \_\_\_\_. \end{aligned}$$

The equation is ________ (true/false), so the system has no solution.

Answer

8. No solution

Guided Solution:
8. 4, false

Suppose we try to solve this system by the elimination method:

$$\begin{aligned} 12y - 8x &= 24 &&\text{Multiplying equation (1) by 4} \\ -12y + 8x &= -24 &&\text{Equation (2)} \\ \hline 0 &= 0. &&\text{Adding, we obtain a true equation.} \end{aligned}$$

We have eliminated both variables, and what remains is a true equation, $0 = 0$. It can be expressed as $0 \cdot x + 0 \cdot y = 0$, and is true for all numbers x and y. If an ordered pair is a solution of one of the original equations, then it will be a solution of the other. The system has an **infinite number of solutions**. The system is consistent, and the equations are dependent.

SPECIAL CASES

When solving a system of two linear equations in two variables:

1. If a false equation is obtained, such as $0 = 7$, then the system has no solution. The system is *inconsistent*, and the equations are *independent*.
2. If a true equation is obtained, such as $0 = 0$, then the system has an infinite number of solutions. The system is *consistent*, and the equations are *dependent*.

Do Exercise 9.

9. Solve by the elimination method:

$$\begin{aligned} 2x - 5y &= 10, \\ -6x + 15y &= -30. \end{aligned}$$

Comparing Methods

We can solve systems of equations graphically, or we can solve them algebraically using substitution or elimination. When deciding which method to use, consider the information in this table as well as directions from your instructor.

METHOD	STRENGTHS	WEAKNESSES
Graphical	Can "see" solutions.	Inexact when solutions involve numbers that are not integers. Solutions may not appear on the part of the graph drawn.
Substitution	Yields exact solutions. Convenient to use when a variable has a coefficient of 1.	Can introduce extensive computations with fractions. Cannot "see" solutions quickly.
Elimination	Yields exact solutions. Convenient to use when no variable has a coefficient of 1. The preferred method for systems of three or more equations in three or more variables. (See Section 3.5.)	Cannot "see" solutions quickly.

Answer

9. Infinitely many solutions

b SOLVING APPLIED PROBLEMS USING ELIMINATION

Let's now solve an applied problem using the elimination method.

EXAMPLE 7 ***Stimulating the Hometown Economy.*** To stimulate the economy in his town of Brewton, Alabama, in 2009, Danny Cottrell, co-owner of The Medical Center Pharmacy, gave each of his full-time employees \$700 and each part-time employee \$300. He asked that each person donate 15% to a charity of his or her choice and spend the rest locally. The money was paid in \$2 bills, a rarely used currency, so that the business community could easily see how the money circulated. Cottrell gave away a total of \$16,000 to his 24 employees. How many full-time employees and how many part-time employees were there?

Source: *The Press-Register,* March 4, 2009

1. **Familiarize.** We let $f =$ the number of full-time employees and $p =$ the number of part-time employees. Each full-time employee received \$700, so a total of $700f$ was paid to them. Similarly, the part-time employees received a total of $300p$. Thus a total of $700f + 300p$ was given away.

2. **Translate.** We translate to two equations.

$$\underbrace{\text{Total amount given away}}_{700f\,+\,300p} \quad \underset{=}{\text{is}} \quad \underset{16{,}000}{\$16{,}000.}$$

$$\underbrace{\text{Total number of employees}}_{f\,+\,p} \quad \underset{=}{\text{is}} \quad \underset{24}{24.}$$

We now have a system of equations:

$$\begin{aligned} 700f + 300p &= 16{,}000, \quad &(1)\\ f + p &= 24. \quad &(2) \end{aligned}$$

3. **Solve.** First, we multiply by -300 on both sides of equation (2) and add:

$$\begin{array}{rcll} 700f + 300p &=& 16{,}000 & \text{Equation (1)}\\ -300f - 300p &=& -7200 & \text{Multiplying by } -300 \text{ on both sides of equation (2)}\\ \hline 400f &=& 8800 & \text{Adding}\\ f &=& 22. & \text{Solving for } f \end{array}$$

Next, we substitute 22 for f in equation (2) and solve for p:

$$\begin{aligned} 22 + p &= 24\\ p &= 2. \end{aligned}$$

4. **Check.** If there are 22 full-time employees and 2 part-time employees, there is a total of $22 + 2$, or 24, employees. The 22 full-time employees received a total of $\$700 \cdot 22$, or \$15,400, and the 2 part-time employees received a total of $\$300 \cdot 2$, or \$600. Then a total of $\$15{,}400 + \$600$, or \$16,000, was given away. The numbers check in the original problem.

5. **State.** There were 22 full-time employees and 2 part-time employees.

Do Exercise 10.

10. ***Bonuses.*** Monica gave each of the full-time employees in her small business a year-end bonus of \$500 while each part-time employee received \$250. She gave a total of \$4000 in bonuses to her 10 employees. How many full-time employees and how many part-time employees did Monica have?

Answer

10. Full-time: 6; part-time: 4

3.3 Exercise Set

For Extra Help
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Reading Check

Choose from the column on the right the word that best completes each sentence. Words may be used more than once.

consistent
inconsistent
dependent
independent

RC1. If a system of equations has a solution, then it is ________________.

RC2. If a system of equations has no solution, then it is ________________.

RC3. If a system of equations has infinitely many solutions, then it is ________________.

RC4. If the graphs of the equations in a system of two equations in two variables are the same line, then the equations are ________________.

RC5. If the graphs of the equations in a system of two equations in two variables are parallel, then the system is ________________.

RC6. If the graphs of the equations in a system of two equations in two variables intersect at one point, then the equations are ________________.

a Solve each system of equations using the elimination method.

1. $x + 3y = 7,$
$-x + 4y = 7$

2. $x + y = 9,$
$2x - y = -3$

3. $9x + 5y = 6,$
$2x - 5y = -17$

4. $2x - 3y = 18,$
$2x + 3y = -6$

5. $5x + 3y = -11,$
$3x - y = -1$

6. $2x + 3y = -9,$
$5x - 6y = -9$

7. $5r - 3s = 19,$
$2r - 6s = -2$

8. $2a + 3b = 11,$
$4a - 5b = -11$

9. $2x + 3y = 1,$
$4x + 6y = 2$

10. $3x - 2y = 1,$
$-6x + 4y = -2$

11. $5x - 9y = 7,$
$7y - 3x = -5$

12. $5x + 4y = 2,$
$2x - 8y = 4$

13. $3x + 2y = 24,$
$2x + 3y = 26$

14. $5x + 3y = 25,$
$3x + 4y = 26$

15. $2x - 4y = 5,$
$2x - 4y = 6$

16. $3x - 5y = -2,$
$5y - 3x = 7$