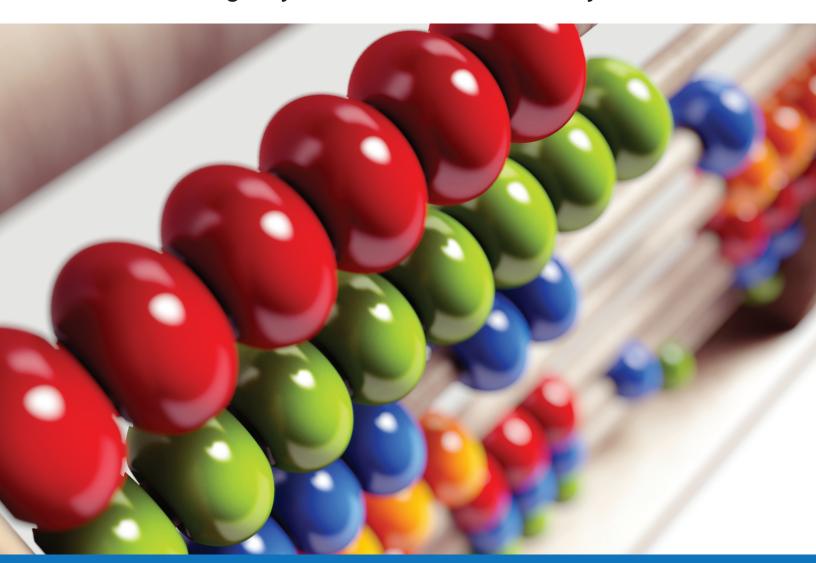


Basic College Mathematics

TWELFTH EDITION

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ALWAYS LEARNING PEARSON

Geometric Formulas

PLANE GEOMETRY

Rectangle

Area: $A = l \cdot w$

Perimeter: $P = 2 \cdot l + 2 \cdot w$

Square

Area: $A = s^2$

Perimeter: $P = 4 \cdot s$

Triangle

Area: $A = \frac{1}{2} \cdot b \cdot h$

Sum of Angle Measures

 $A + B + C = 180^{\circ}$

Right Triangle

Pythagorean Theorem:

 $a^2 + b^2 = c^2$

Parallelogram

Area: $A = b \cdot h$

Trapezoid

Area: $A = \frac{1}{2} \cdot h \cdot (a + b)$

Circle

Area: $A = \pi \cdot r^2$

Circumference:

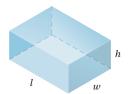
 $C = \pi \cdot d = 2 \cdot \pi \cdot r \left(\frac{22}{7}\right)$ and 3.14

are different approximations for π)

SOLID GEOMETRY

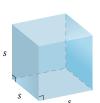
Rectangular Solid

Volume: $V = l \cdot w \cdot h$



Cube

Volume: $V = s^3$



Volume: $V = \pi \cdot r^2 \cdot h$

Surface Area:

 $S = 2 \cdot \pi \cdot r \cdot h + 2 \cdot \pi \cdot r^2$

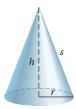


Right Circular Cone

Volume: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

Surface Area:

 $S = \pi \cdot r^2 + \pi \cdot r \cdot s$



Sphere

Volume: $V = \frac{4}{3} \cdot \pi \cdot r^3$ Surface Area: $S = 4 \cdot \pi \cdot r^2$



Fraction, Decimal, and **Percent Equivalents**

| Fraction Notation | $\frac{1}{10}$ | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{3}{10}$ | $\frac{1}{3}$ | $\frac{3}{8}$ | 2 5 | $\frac{1}{2}$ | 3 5 | <u>5</u> 8 | $\frac{2}{3}$ | $\frac{7}{10}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | $\frac{5}{6}$ | $\frac{7}{8}$ | $\frac{9}{10}$ | $\frac{1}{1}$ |
|----------------------|----------------|---------------------|---------------------|---------------|---------------|----------------|---------------------|---------------------|--------|---------------|--------|----------------------------|---|----------------|---------------|---------------|---------------------|---------------------|----------------|---------------|
| Decimal Notation | 0.1 | 0.125 | $0.16\overline{6}$ | 0.2 | 0.25 | 0.3 | 0.333 | 0.375 | 0.4 | 0.5 | 0.6 | 0.625 | $0.66\overline{6}$ | 0.7 | 0.75 | 8.0 | $0.83\overline{3}$ | 0.875 | 0.9 | 1 |
| Percent Notation | 10% | 12.5% or 12½% | 16.6% or 163% | | 25% | 30% | 33.3% or 333% | 37.5% or 37½% | 40% | 50% | 60% | 62.5% or $62\frac{1}{2}\%$ | 66.6% or 66 ² ₃ % | 70% | 75% | 80% | 83.3% or 83½% | 87.5% or 87½% | 90% | 100% |



Recall that to compute an average, we add the numbers and then divide the sum by the number of addends. We have

$$\frac{7\frac{5}{8} + 8\frac{1}{4} + 8\frac{3}{4} + 9\frac{1}{8} + 10\frac{1}{2}}{5} = \frac{7\frac{5}{8} + 8\frac{2}{8} + 8\frac{6}{8} + 9\frac{1}{8} + 10\frac{4}{8}}{5}$$

$$= \frac{42\frac{18}{8}}{5} = \frac{42\frac{9}{4}}{5} = \frac{\frac{177}{4}}{5} \qquad \begin{array}{c} \text{Adding, simplifying, and converting to fraction notation} \\ = \frac{177}{4} \cdot \frac{1}{5} & \text{Multiplying by the reciprocal} \\ = \frac{177}{20} = 8\frac{17}{20}. & \text{Converting to a mixed numeral} \end{array}$$



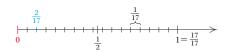
The average length of the logs is $8\frac{17}{20}$ ft.

Do Exercises 9–11. ▶

C ESTIMATION WITH FRACTION NOTATION AND MIXED NUMERALS

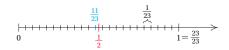
EXAMPLES Estimate each of the following as $0, \frac{1}{2}$, or 1.

8.
$$\frac{2}{17}$$



A fraction is close to 0 when the numerator is small in comparison to the denominator. Since 2 is small in comparison to 17, we have $\frac{2}{17} \approx 0$.

9.
$$\frac{11}{23}$$



A fraction is close to $\frac{1}{2}$ when the denominator is about twice the numerator. Twice the numerator of this fraction is 22, and 22 is close to 23. Thus, $\frac{11}{23} \approx \frac{1}{2}$.

10.
$$\frac{43}{41}$$

A fraction is close to 1 when the numerator is nearly equal to the denominator. Since 43 is nearly equal to 41, we have $\frac{43}{41}\approx 1$.

Do Exercises 12–15. ▶

EXAMPLE 11 Estimate $16\frac{8}{9}+11\frac{2}{13}-4\frac{22}{43}$ by estimating each mixed numeral as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$. We estimate each mixed numeral. Then we calculate:

$$16\frac{8}{9} + 11\frac{2}{13} - 4\frac{22}{43} \approx 17 + 11 - 4\frac{1}{2}$$
$$= 28 - 4\frac{1}{2} = 23\frac{1}{2}.$$

Do Exercises 16–18. ▶

- 9. Rachel has triplets. Their birth weights are $3\frac{1}{2}$ lb, $2\frac{3}{4}$ lb, and $3\frac{1}{8}$ lb. What is the average weight of her babies?
- 10. Find the average of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$.
- 11. Find the average of $\frac{3}{4}$ and $\frac{4}{5}$.

Estimate each of the following as 0, $\frac{1}{2}$, or 1.

12.
$$\frac{3}{59}$$

13.
$$\frac{61}{59}$$

14.
$$\frac{29}{59}$$

15.
$$\frac{57}{59}$$

Estimate each of the following by estimating each mixed numeral as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$ and estimating each fraction as $0, \frac{1}{2}$, or 1.

16.
$$5\frac{9}{10} + 26\frac{1}{2} - 10\frac{3}{29}$$

17.
$$10\frac{7}{8} \cdot \left(25\frac{11}{13} - 14\frac{1}{9}\right)$$

18.
$$\left(10\frac{4}{5} + 7\frac{5}{9}\right) \div \frac{17}{30}$$

Answers

9.
$$3\frac{1}{8}$$
 lb 10. $\frac{5}{9}$ 11. $\frac{31}{40}$ 12. 0 13. 1

14.
$$\frac{1}{2}$$
 15. 1 16. $22\frac{1}{2}$ 17. 132 18. 37

3.7

Exercise Set

For Extra Help MyMathLab®









Reading Check

Match the beginning of each statement with the correct ending from the list at the right so that the rules for order of operations are listed in the correct order.

1.
$$\frac{5}{8} \div \frac{1}{4} - \frac{2}{3} \cdot \frac{4}{5}$$

2.
$$\frac{4}{7} \cdot \frac{7}{15} + \frac{2}{3} \div 8$$

3.
$$\frac{3}{4} - \frac{2}{3} \cdot \left(\frac{1}{2} + \frac{2}{5}\right)$$

4.
$$\frac{3}{4} \div \frac{1}{2} \cdot \left(\frac{8}{9} - \frac{2}{3} \right)$$

5.
$$28\frac{1}{8} - 5\frac{1}{4} + 3\frac{1}{2}$$

6.
$$10\frac{3}{5} - 4\frac{1}{10} - 1\frac{1}{2}$$

7.
$$\frac{7}{8} \div \frac{1}{2} \cdot \frac{1}{4}$$

8.
$$\frac{7}{10} \cdot \frac{4}{5} \div \frac{2}{3}$$

9.
$$\left(\frac{2}{3}\right)^2 - \frac{1}{3} \cdot 1\frac{1}{4}$$

10.
$$\left(\frac{3}{4}\right)^2 + 3\frac{1}{2} \div 1\frac{1}{4}$$

11.
$$\frac{1}{2} - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

12.
$$1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^3$$

13.
$$\left(\frac{2}{3} + \frac{3}{4}\right) \div \left(\frac{5}{6} - \frac{1}{3}\right)$$

14.
$$\left(\frac{3}{5} - \frac{1}{2}\right) \div \left(\frac{3}{4} - \frac{3}{10}\right)$$

15.
$$\left(\frac{1}{2} + \frac{1}{3}\right)^2 \cdot 144 - \frac{5}{8} \div 10\frac{1}{2}$$

16.
$$\left(3\frac{1}{2}-2\frac{1}{3}\right)^2+6\cdot 2\frac{1}{2} \div 32$$

b Simplify.

17.
$$\frac{\frac{3}{8}}{\frac{11}{8}}$$

18.
$$\frac{\frac{1}{8}}{\frac{3}{4}}$$

19.
$$\frac{4}{\frac{6}{7}}$$

20.
$$\frac{\frac{3}{8}}{12}$$

21.
$$\frac{\frac{1}{40}}{\frac{1}{50}}$$

22.
$$\frac{\frac{7}{9}}{\frac{3}{9}}$$

23.
$$\frac{\frac{1}{10}}{10}$$

24.
$$\frac{28}{\frac{7}{4}}$$

25.
$$\frac{\frac{5}{18}}{1\frac{2}{3}}$$

26.
$$\frac{2\frac{1}{5}}{\frac{7}{10}}$$

27.
$$\frac{\frac{5}{9} - \frac{1}{6}}{\frac{2}{3}}$$

28.
$$\frac{\frac{7}{12}}{\frac{5}{8} - \frac{1}{4}}$$

29.
$$\frac{\frac{3}{8} - \frac{1}{4}}{\frac{7}{8} - \frac{1}{2}}$$

30.
$$\frac{\frac{3}{5} - \frac{1}{2}}{\frac{1}{2} - \frac{2}{5}}$$

31. Find the average of
$$\frac{2}{3}$$
 and $\frac{7}{8}$.

32. Find the average of
$$\frac{1}{4}$$
 and $\frac{1}{5}$.

33. Find the average of
$$\frac{1}{6}$$
, $\frac{1}{8}$, and $\frac{3}{4}$.

34. Find the average of
$$\frac{4}{5}$$
, $\frac{1}{2}$, and $\frac{1}{10}$.

35. Find the average of
$$3\frac{1}{2}$$
 and $9\frac{3}{8}$.

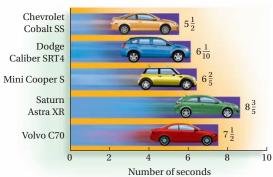
36. Find the average of
$$10\frac{2}{3}$$
 and $24\frac{5}{6}$.

- 37. Hiking the Appalachian Trail. Ellen camped and hiked for three consecutive days along a section of the Appalachian Trail. The distances she hiked on the three days were $15\frac{5}{32}$ mi, $20\frac{3}{16}$ mi, and $12\frac{7}{8}$ mi. Find the average of these distances.
- **38.** *Vertical Leaps.* Eight-year-old Zachary registered vertical leaps of $12\frac{3}{4}$ in., $13\frac{3}{4}$ in., $13\frac{1}{2}$ in., and 14 in. Find his average vertical leap.
- **39.** Black Bear Cubs. Black bears typically have two cubs. In January 2007 in northern New Hampshire, a black bear sow gave birth to a litter of 5 cubs. This is so rare that Tom Sears, a wildlife photographer, spent 28 hr per week for six weeks watching for the perfect opportunity to photograph this family of six. At the time of this photo, an observer estimated that the cubs weighed $7\frac{1}{2}$ lb, 8 lb, $9\frac{1}{2}$ lb, $10\frac{5}{8}$ lb, and $11\frac{3}{4}$ lb. What was the average weight of the cubs?
- **40.** *Acceleration.* The results of a *Road & Track* road acceleration test for five cars are given in the graph below. The test measures the time in seconds required to go from 0 mph to 60 mph. What was the average time?

Source: Andrew Timmins, New Hampshire Fish and Game Department, *Northcountry News*, Warren, NH; Tom Sears, photographer

Acceleration: 0 mph to 60 mph





SOURCE: Road & Track, October 2008, pp.156-157

41.
$$\frac{2}{47}$$

42.
$$\frac{5}{12}$$

43.
$$\frac{7}{8}$$

44.
$$\frac{1}{13}$$

45.
$$\frac{6}{11}$$

46.
$$\frac{11}{13}$$

47.
$$\frac{7}{15}$$

48.
$$\frac{1}{16}$$

48.
$$\frac{1}{16}$$
 49. $\frac{7}{100}$ **50.** $\frac{5}{9}$

50.
$$\frac{5}{9}$$

51.
$$\frac{19}{20}$$

52.
$$\frac{4}{5}$$

Estimate each of the following by estimating each mixed numeral as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$ and by estimating each fraction as 0, $\frac{1}{2}$, or 1.

53.
$$2\frac{7}{8}$$

54.
$$12\frac{5}{6}$$

55.
$$\frac{4}{5} + \frac{7}{8}$$

56.
$$\frac{1}{12} \cdot \frac{7}{15}$$

57.
$$24 \div 7\frac{8}{9}$$

58.
$$43\frac{16}{17} \div 11\frac{2}{13}$$

59.
$$7\frac{29}{60} + 10\frac{12}{13} \cdot 24\frac{2}{17}$$

60.
$$1\frac{5}{8} + 1\frac{27}{28} \cdot 6\frac{35}{74}$$

61.
$$16\frac{1}{5} \div 2\frac{1}{11} + 25\frac{9}{10} - 4\frac{11}{23}$$
 62. $96\frac{2}{13} \div 5\frac{19}{20} + 3\frac{1}{7} \cdot 5\frac{18}{21}$

62.
$$96\frac{2}{13} \div 5\frac{19}{20} + 3\frac{1}{7} \cdot 5\frac{18}{21}$$

Skill Maintenance

Simplify.

63.
$$12 + 30 \div 3 - 2$$
 [1.9c]

65.
$$10^2 - [3 \cdot 2^4 \div (10 - 2) + 5 \cdot 2]$$
 [1.9d]

64.
$$5 \cdot 2^2 \div 10$$
 [1.9c]

66.
$$(10 + 3 \cdot 4 \div 6)^2 - 11 \cdot 2^2$$
 [1.9c]

Synthesis

224

71. In the sum below, a and b are digits. Find a and b.

$$\frac{a}{17} + \frac{1b}{23} = \frac{35a}{391}$$

73. Consider only the numbers 2, 3, 4, and 5. Assume each is placed in a blank in the following.

What placement of the numbers in the blanks yields the largest sum?

72. Consider only the numbers 3, 4, 5, and 6. Assume each can be placed in a blank in the following.

What placement of the numbers in the blanks yields the largest sum?

74. Use a calculator to arrange the following in order from smallest to largest.

$$\frac{3}{4}$$
, $\frac{17}{21}$, $\frac{13}{15}$, $\frac{7}{9}$, $\frac{15}{17}$, $\frac{13}{12}$, $\frac{19}{22}$

Summary and Review

Vocabulary Reinforcement

Complete each statement with the correct term from the given list. Some of the choices may not be used and some may be used more than once.

- 1. The ______ of two numbers is the smallest number that is a multiple of both numbers. [3.1a]
- 2. A ______ represents a sum of a whole number and a fraction less than 1. [3.4a]
- **3.** To multiply using mixed numerals, we first convert to _____ notation. [3.6a]
- 4. A _____ contains a fraction in its numerator and/or denominator. [3.7b]
- **5.** To add fractions, the ______ of the fractions must be the same. [3.2a]
- **6.** The least common denominator of two fractions is the ______ of the denominators of the fractions. [3.2a]
- 7. When finding the LCM of a set of numbers using prime factorizations, we use each prime number the ______ number of times that it appears in any one factorization. [3.1a]
- **8.** To compare two fractions with a common denominator, we compare their _______. [3.3b]

greatest

least

numerators

denominators

fraction

decimal

mixed numeral

complex fraction

least common multiple

greatest common factor

Concept Reinforcement

Determine whether each statement is true or false.

- 1. The mixed numeral $5\frac{2}{3}$ can be represented by the sum $5 \cdot \frac{3}{3} + \frac{2}{3}$. [3.4a]
 - **2.** The least common multiple of two numbers is always larger than or equal to the larger number. [3.1a]
 - ____ 3. The sum of any two mixed numerals is a mixed numeral. [3.5a]
 - **4.** The product of any two mixed numerals is greater than 1. [3.6a]

Study Guide

Objective 3.1a Find the least common multiple, or LCM, of two or more numbers.

Example Find the LCM of 105 and 90.

$$105 = 3 \cdot 5 \cdot 7,$$

$$90 = 2 \cdot 3 \cdot 3 \cdot 5;$$

$$LCM = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 630$$

Practice Exercise

1. Find the LCM of 52 and 78.

Objective 3.2a Add using fraction notation.

Example Add:
$$\frac{5}{24} + \frac{7}{45}$$
.

$$\frac{5}{24} + \frac{7}{45} = \frac{5}{2 \cdot 2 \cdot 2 \cdot 3} \cdot \frac{3 \cdot 5}{3 \cdot 5} + \frac{7}{3 \cdot 3 \cdot 5} \cdot \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$$

$$= \frac{75}{360} + \frac{56}{360} = \frac{131}{360}$$

Practice Exercise

2. Add:
$$\frac{19}{60} + \frac{11}{36}$$
.

Objective 3.3a Subtract using fraction notation.

Example Subtract:
$$\frac{7}{12} - \frac{11}{60}$$
.

$$\frac{7}{12} - \frac{11}{60} = \frac{7}{12} \cdot \frac{5}{5} - \frac{11}{60} = \frac{35}{60} - \frac{11}{60}$$

$$= \frac{35 - 11}{60} = \frac{24}{60} = \frac{2 \cdot \cancel{12}}{5 \cdot \cancel{12}} = \frac{2}{5}$$

Practice Exercise

3. Subtract:
$$\frac{29}{35} - \frac{5}{7}$$
.

Objective 3.3b Use < or > with fraction notation to write a true sentence.

Example Use < or > for \square to write a true sentence: $\frac{5}{12} \square \frac{9}{16}$.

Writing both fractions with the LCD, 48, we have

$$\frac{20}{48} \, \square \, \frac{27}{48}.$$

Since 20 < 27, $\frac{20}{48} < \frac{27}{48}$ and thus $\frac{5}{12} < \frac{9}{16}$.

Practice Exercise

4. Use < or > for \square to write a true sentence:

$$\frac{3}{13} \square \frac{5}{12}$$
.

Objective 3.3c Solve equations of the type x + a = b and a + x = b, where a and b may be fractions.

Example Solve: $x + \frac{1}{6} = \frac{5}{8}$.

$$x + \frac{1}{6} = \frac{5}{8}$$

$$x + \frac{1}{6} - \frac{1}{6} = \frac{5}{8} - \frac{1}{6}$$

$$x = \frac{5}{8} \cdot \frac{3}{3} - \frac{1}{6} \cdot \frac{4}{4} = \frac{15}{24} - \frac{4}{24} = \frac{11}{24}$$

Practice Exercise

5. Solve: $\frac{2}{9} + x = \frac{9}{11}$.

Objective 3.4a Convert between mixed numerals and fraction notation.

Example Convert $2\frac{5}{13}$ to fraction notation: $2\frac{5}{13} = \frac{31}{13}$

Example Convert $\frac{40}{9}$ to a mixed numeral: $\frac{40}{9} = 4\frac{4}{9}$.

Practice Exercises

- **6.** Convert $8\frac{2}{3}$ to fraction notation.
- 7. Convert $\frac{47}{6}$ to a mixed numeral.