



Pearson New International Edition

Quality  
Donna C. Summers  
Fifth Edition

# Pearson New International Edition

---

Quality  
Donna C. Summers  
Fifth Edition

Several types of variation are tracked with statistical methods. These include:

1. Within-piece variation, or the variation within a single item or surface. For example, a single square yard of fabric may be examined to see if the color varies from one location to another.
2. Piece-to-piece variation, or the variation that occurs among pieces produced at approximately the same time. For example, in a production run filling gallon jugs with milk, when each of the milk jugs is checked after the filling station, the fill level from jug to jug will be slightly different.
3. Time-to-time variation, or the variation in the product produced at different times of the day—for example, the comparison of a part that has been stamped at the beginning of a production run with the part stamped at the end of a production run.

The variation in a process is studied by sampling the process. Samples are grouped into subgroups depending on whether the variation under study is piece-to-piece variation, within-piece variation, or time-to-time variation. The groupings of samples for piece-to-piece variation depend on the number of products analyzed during a particular time period. Within-piece variation samples are arranged in subgroups according to where in the process they are taken from. Samples for time-to-time variation are grouped according to the time of day that they were taken from the process.

## Chance and Assignable Causes

Even in the most precise process no two parts are exactly alike. Whether these differences are due to chance causes in the process or to assignable causes needs to be determined. On the one hand, *chance*, or *common causes* are small random changes in the process that cannot be avoided. These small differences are due to the inherent variation present in all processes. They consistently affect the process and its performance day after day, every day. Variation of this type is only removable by making a change in the existing process. Removing chance causes from a system usually involves management intervention.

*Assignable causes*, on the other hand, are variations in the process that can be identified as having a specific cause. Assignable causes are causes that are not part of the process on a regular basis. This type of variation arises because of specific circumstances. It is these circumstances that the quality assurance analyst seeks to find. Examples of assignable causes include a size change in a part that occurs when chips build up around a work-holding device in a machining operation, changes in the thickness of incoming raw material, or a broken tool. Sources of variation can be found in the process itself, the materials used, the operator's actions, or the environment. Examples of factors that can contribute to process variation include tool wear, machine vibration, and work-holding devices. Changes in

material thickness, composition, or hardness are sources of variation. Operator actions affecting variation include overadjusting the machine, making an error during the inspection activity, changing the machine settings, or failing to properly align the part before machining. Environmental factors affecting variation include heat, light, radiation, and humidity.

To illustrate chance and assignable causes consider the following example.

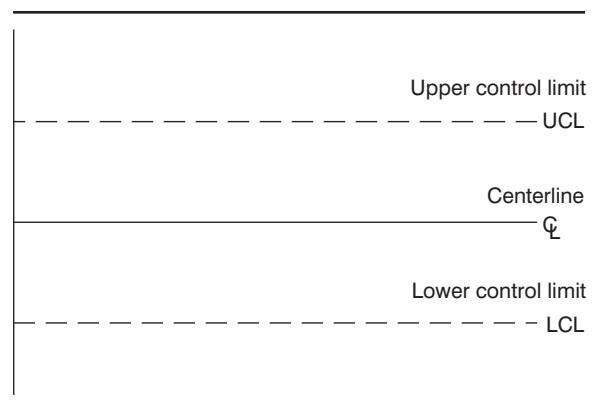
### Example 2 Chance Causes and Assignable Causes

To create the flooring of an automobile, a flexible fiberglass form is affixed to carpet. This operation takes place in an oven and requires intense heat. To achieve a firm bond between the two materials, the temperature in the oven must remain within a very narrow range. RYMAX, a manufacturer of automobile flooring, operates several of these ovens in one area. The common causes, those causes affecting all of the ovens, are the temperature and humidity levels in the building. Special causes affect only certain ovens. Special causes would include leaving the oven door open to free a trapped form or a door that doesn't seal tightly. Can you think of other common or special causes for these ovens?



## CONTROL CHARTS FOR VARIABLES

To create a control chart, samples, arranged into subgroups, are taken during the process. The averages of the subgroup values are plotted on the control chart. The *centerline (CL)* of this chart shows where the process average is centered, the central tendency of the data. The *upper control limit (UCL)* and *lower control limit (LCL)*, calculated based on  $\pm 3\sigma$ , describe the spread of the process (Figure 4). Once the chart is constructed, it presents the user with a picture of what the process is currently capable of producing. In other words, we can expect future production to fall between these  $\pm 3\sigma$  limits 99.73 percent of the time, providing the process does not change and is under control.



**FIGURE 4** Control Chart Showing Centerline and Control Limits

Since control charts show changes in the process measurements, they allow for early detection of process changes. Instead of waiting until an entire production run is complete or until the product reaches the end of the assembly line, management can have critical part dimensions checked and charted throughout the process. If a part or group of parts has been made incorrectly, production can be stopped, adjusted, or otherwise modified to produce parts correctly. This approach permits corrections to be made to the process before a large number of parts is produced or, in some cases, before the product exceeds the specifications. Early detection can avoid scrap, rework, unnecessary adjustments to the process, and/or production delays.

*Variables* are the measurable characteristics of a product or service. Examples of variables include the height, weight, or length of a part. One of the most commonly used variable chart combinations in statistical process control is the  $\bar{X}$  and R charts. Typical  $\bar{X}$  and R charts are shown in Figure 5.  $\bar{X}$  and R charts are used together to determine the distribution of the subgroup averages of sample measurements taken from a process. The importance of using these two charts in conjunction with each other will become apparent shortly.

## $\bar{X}$ AND R CHARTS

The  $\bar{X}$  chart is used to monitor the variation of the subgroup averages that are calculated from the individual sampled data. Averages rather than individual observations are used on control charts because average values will indicate a change in the amount of variation much faster than will individual values. Control limits on this chart are used to evaluate the variation from one subgroup to another. A control chart is constructed using the following steps:

- Step 1.** Define the problem.
- Step 2.** Select the quality characteristic to be measured.
- Step 3.** Choose a rational subgroup size to be sampled.
- Step 4.** Collect the data.
- Step 5.** Determine the trial centerline for the  $\bar{X}$  chart.
- Step 6.** Determine the trial control limits for the  $\bar{X}$  chart.
- Step 7.** Determine the trial control limits for the R chart.
- Step 8.** Examine the process: control chart interpretation.
- Step 9.** Revise the chart.
- Step 10.** Achieve the purpose.

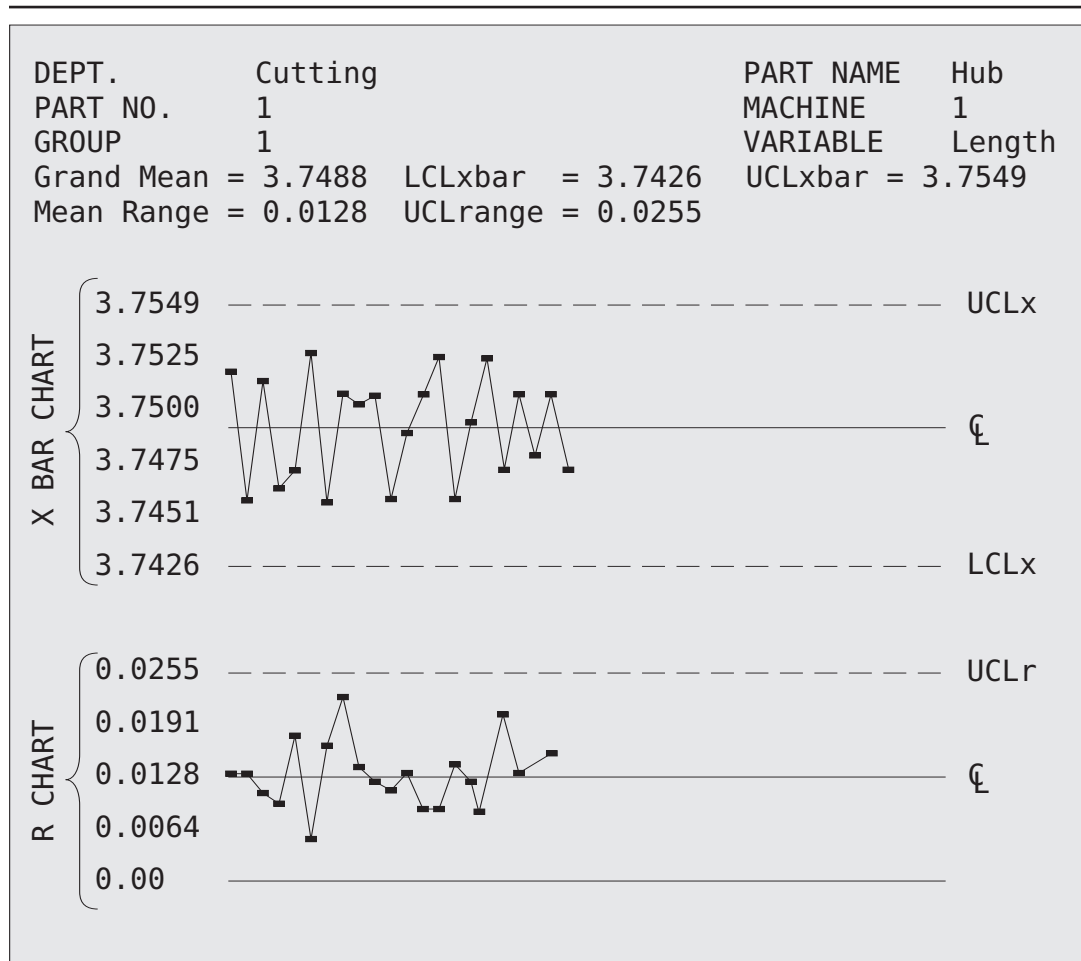


FIGURE 5 Typical  $\bar{X}$  and R Chart



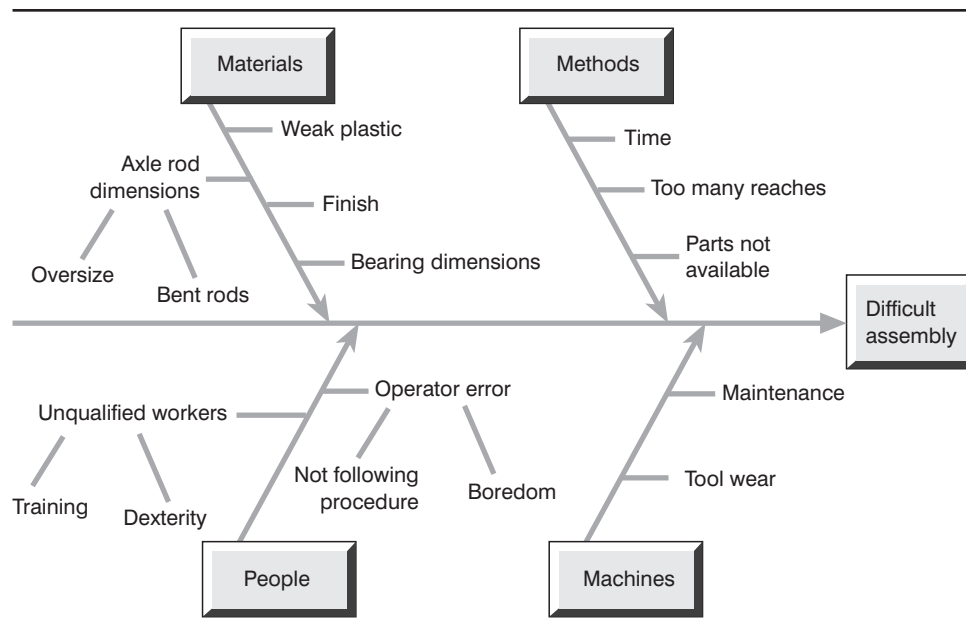


FIGURE 6 Assembly: Cause-and-Effect Diagram

The following steps and examples explain the construction of an  $\bar{X}$  chart.

## 1. Define the Problem

In any situation it is necessary to determine what the goal of monitoring a particular quality characteristic or group of characteristics is. To merely say, “Improve quality,” is not enough. Nor is it sufficient to say, “We would like to see fewer parts out of specification.” Out of which specification? Is it total product performance that is being affected or just one particular dimension? Sometimes several aspects of a part are critical for part performance; occasionally only one is. It is more appropriate to say, “The length of these parts appears to be consistently below the lower specification limit. This causes the parts to mate incorrectly. Why are these parts below specification and how far below are they?” In the second statement we have isolated the length of the part as a critical dimension. From here, control charts can be placed on the process to help determine where the true source of the problem is located.

### Example 3 JRPS Assembly: Defining the Problem

JRPS's assembly area has been experiencing serious delays in the assembly of shafts and armatures. As quality assurance manager, you have been asked to determine the cause of these delays and fix the problems as soon as possible. To best utilize the limited time available, you convene a meeting involving those closest to the assembly problems. Representatives from production, supervision, manufacturing engineering, industrial engineering, quality assurance, and maintenance have been able to generate a variety of possible problems. During this meeting, a cause-and-effect diagram was created,

showing the potential causes for the assembly difficulties (Figure 6). Discussions during the meeting revealed that the shaft dimensions could be the major cause of assembly problems.



## 2. Select the Quality Characteristic to Be Measured

Variable control charts are based on measurements. The characteristics selected for measurement should be ones that affect product or service performance. Choice of a characteristic to be measured depends on what is being investigated. Characteristic choice also depends on whether the process is being monitored for within-piece variation, piece-to-piece variation, or variation over time. Product or service characteristics such as length, height, viscosity, color, temperature, and velocity are typically used in manufacturing settings. Characteristics affecting performance are found in many aspects of a product, including raw materials, components, subassemblies, and finished products. Delivery times, checkout times, and service times are examples of characteristics chosen in a service industry. It is crucial to identify important characteristics; avoid the tendency to try to establish control charts for all measurements.

### Example 4 JRPS Assembly: Identifying the Quality Characteristic

As the troubleshooting meeting described in Example 3 continues, further investigation reveals that the length of the shaft is hindering assembly operations. The characteristic to measure has been identified as piece-to-piece variation in the length of the shafts. To begin to study the situation, measurements of the lengths of the shafts will be sampled.



### 3. Choose a Rational Subgroup Size to Be Sampled

Subgroups, and the samples composing them, must be homogeneous. A *homogeneous subgroup* will have been produced under the same conditions, by the same machine, the same operator, the same mold, and so on. Homogeneous lots can also be designated by equal time intervals. Samples should be taken in an unbiased, random fashion. They should be representative of the entire population. Subgroup formation should reflect the type of variation under study. Subgroups used in investigating piece-to-piece variation will not necessarily be constructed in the same manner as subgroups formed to study time-to-time variation. The letter  $n$  is used to designate the number of samples taken within a subgroup. When constructing  $\bar{X}$  and  $R$  charts, keep the subgroup sample size constant for each subgroup taken.

Decisions concerning the specific size of the subgroup— $n$ , or the number of samples—require judgment. Sampling should occur frequently enough to detect changes in the process. Ask how often is the system expected to change. Examine the process and identify the factors causing change in the process. To be effective, sampling must occur as often as the system's most frequently changing factor. Once the number and frequency of sampling have been selected, they should not be changed unless the system itself has changed.

Realistically, sampling frequency must balance the value of the data obtained with the costs of taking the samples. Sampling is usually more frequent when control charts are first used to monitor the process. As process improvements are made and the process stabilizes, the frequency of sampling and subgroup size can be decreased.

When gathering sample data, it is important to have the following information in order to properly analyze the data:

1. *Who* will be collecting the data?
2. *What* aspect of the process is to be measured?
3. *Where* or at what point in the process will the sample be taken?
4. *When* or how frequently will the process be sampled?
5. *Why* is this particular sample being taken?
6. *How* will the data be collected?
7. *How many* samples will be taken (subgroup size)?

Some other guidelines to be followed include:

- The larger the subgroup size, the more sensitive the chart becomes to small variations in the process average. This will provide a better picture of the process since it allows the investigator to detect changes in the process quickly.
- While a larger subgroup size makes for a more sensitive chart, it also increases inspection costs.

- Destructive testing may make large subgroup sizes unfeasible. For example, it would not make sense for a fireworks manufacturer to test each and every one of its products.
- Subgroup sizes smaller than four do not create a representative distribution of subgroup averages. Subgroup averages are nearly normal for subgroups of four or more even when sampled from a nonnormal population.
- When the subgroup size exceeds 10, the standard deviation ( $s$ ) chart, rather than the range ( $R$ ) chart, should be used. For large subgroup sizes, the  $s$  chart gives a better representation of the true dispersion or true differences between the individuals sampled than does the  $R$  chart.

#### Example 5 JRPS Assembly: Selecting Subgroup Sample Size

The production from the machine making the shafts first looked at in Example 3 is consistent at 150 per hour. Since the process is currently exhibiting problems, your team has decided to take a sample of five measurements every 10 minutes from the production. The values for the day's production run are shown in Figure 7.



### 4. Collect the Data

To create a control chart, an amount of data sufficient to accurately reflect the statistical control of the process must be gathered. A minimum of 20 subgroups of sample size  $n = 4$  is suggested. Each time a subgroup of sample size  $n$  is taken, an average is calculated for the subgroup. To do this, the individual values are recorded, summed, and then divided by the number of samples in the subgroup. This average,  $\bar{X}_i$ , is then plotted on the control chart.

#### Example 6 JRPS Assembly: Collecting Data

A sample of size  $n = 5$  is taken at 10-minute intervals from the process making shafts. As shown in Figure 7, a total of 21 subgroups of sample size  $n = 5$  have been taken. Each time a subgroup sample is taken, the individual values are recorded [Figure 7, (1)], summed, and then divided by the number of samples taken to get the average for the subgroup [Figure 7, (2)]. This subgroup average is then plotted on the control chart [Figure 8, (1)].

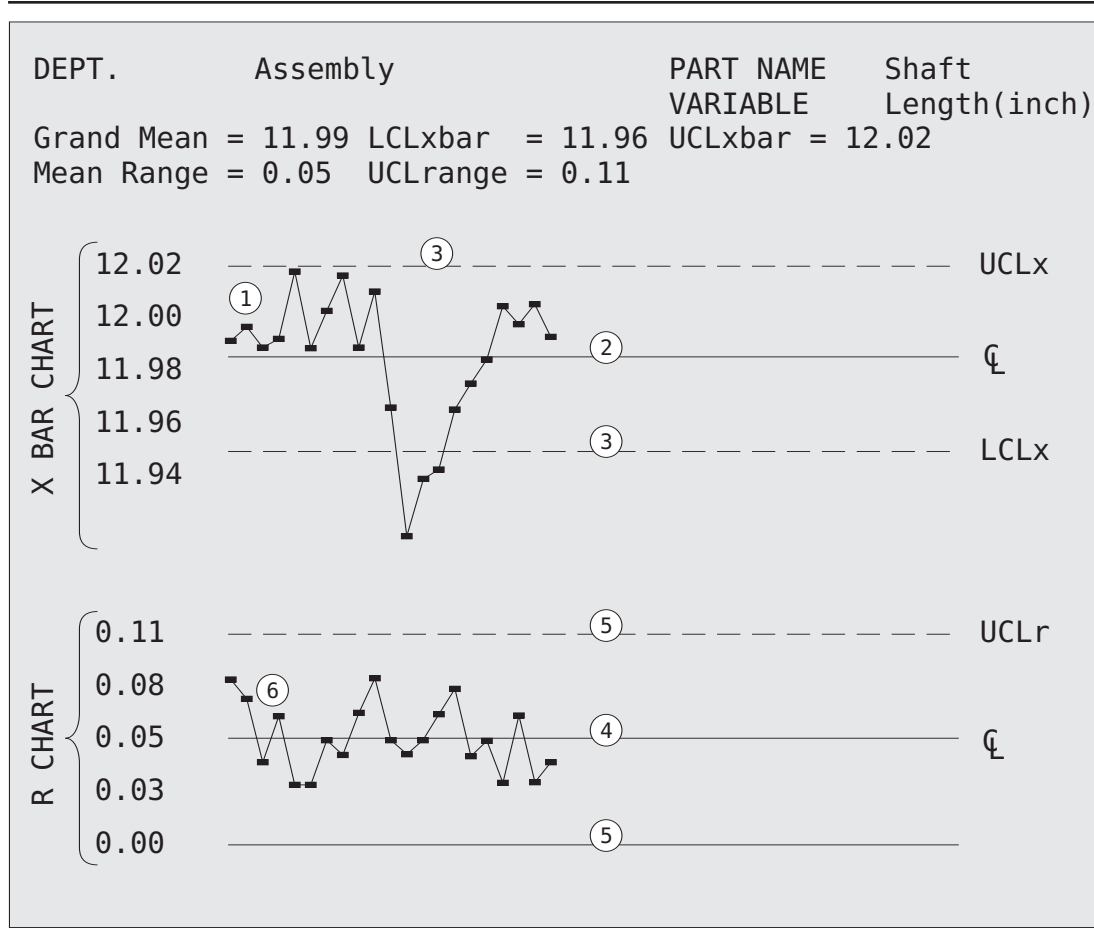


### 5. Determine the Trial Centerline for the $\bar{X}$ Chart

The centerline of the control chart is the process average. It would be the mean,  $\mu$ , if the average of the population measurements for the entire process were known. Since the value of the population mean  $\mu$  cannot be determined unless all of the parts being produced are measured, in its place the grand average of the subgroup averages,  $\bar{\bar{X}}$  ( $X$  double bar), is used. The grand average, or  $\bar{\bar{X}}$ , is calculated by summing all the subgroup averages and then dividing by the

DEPT. PART NO. GROUP	Assembly 1 1		PART NAME MACHINE VARIABLE	Shaft 1 length	
Subgroup	1	2	3	4	5
Time	07:30	07:40	07:50	08:00	08:10
Date	07/02/95	07/02/95	07/02/95	07/02/95	07/02/95
1	11.95	12.03	12.01	11.97	12.00
2	12.00	12.02	12.00	11.98	12.01
3	12.03	11.96	11.97	12.00	12.02
4	11.98	12.00	11.98	12.03	12.03
5	12.01	11.98	12.00	11.99	12.02
$\bar{X}$	11.99	12.00	11.99	11.99	12.02
Range	0.08	0.07	0.04	0.06	0.03
Subgroup	6	7	8	9	10
Time	08:20	08:30	08:40	08:50	09:00
Date	07/02/95	07/02/95	07/02/95	07/02/95	07/02/95
1	11.98	12.00	12.00	12.00	12.02
2	11.98	12.01	12.01	12.02	12.00
3	12.00	12.03	12.04	11.96	11.97
4	12.01	12.00	12.00	12.00	12.05
5	11.99	11.98	12.02	11.98	12.00
$\bar{X}$	11.99	12.00	12.01	11.99	12.01
Range	0.03	0.05	0.04	0.06	0.08
Subgroup	11	12	13	14	15
Time	09:10	09:20	09:30	09:40	09:50
Date	07/02/95	07/02/95	07/02/95	07/02/95	07/02/95
1	11.98	11.92	11.93	11.99	12.00
2	11.97	11.95	11.95	11.93	11.98
3	11.96	11.92	11.98	11.94	11.99
4	11.95	11.94	11.94	11.95	11.95
5	12.00	11.96	11.96	11.96	11.93
$\bar{X}$	11.97	11.94	11.95	11.95	11.97
Range	0.05	0.04	0.05	0.06	0.07
Subgroup	16	17	18	19	20
Time	10:00	10:10	10:20	10:30	10:40
Date	07/02/95	07/02/95	07/02/95	07/02/95	07/02/95
1	12.00	12.02	12.00	11.97	11.99
2	11.98	11.98	12.01	12.03	12.01
3	11.99	11.97	12.02	12.00	12.02
4	11.96	11.98	12.01	12.01	12.00
5	11.97	11.99	11.99	11.99	12.01
$\bar{X}$	11.98	11.99	12.01	12.00	12.01
Range	0.04	0.05	0.03	0.06	0.03
		$R = 0.05 = 12.02 - 11.97$			
Subgroup	21	$\frac{12.00 + 11.98 + 11.99 + 11.96 + 11.97}{5} = 11.98$			
Time	10:50				
Date	07/02/95				
1	12.00				
2	11.98				
3	11.99				
4	11.99				
5	12.02				
$\bar{X}$	12.00				
Range	0.04				

FIGURE 7 Values for a Day's Production

FIGURE 8  $\bar{X}$  and R Control Charts for Shaft Length

number of subgroups. This value is plotted as the centerline of the  $\bar{X}$  chart:

$$\bar{\bar{X}} = \frac{\sum_{i=1}^m \bar{X}_i}{m}$$

where

$\bar{\bar{X}}$  = average of the subgroup averages

$\bar{X}_i$  = average of the  $i$ th subgroup

$m$  = number of subgroups

## 6. Determine the Trial Control Limits for the $\bar{X}$ Chart

We learned that 99.73 percent of the data under a normal curve falls within Figure 1 showed how a control chart is a time-dependent pictorial representation of a normal curve displaying the distribution of the averages of the samples taken from the process.

Because of this, control limits are established at  $\pm 3$  standard deviations from the centerline for the process using the following formulas:

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}}$$

where

UCL = upper control limit of the  $\bar{X}$  chart

LCL = lower control limit of the  $\bar{X}$  chart

$\sigma_{\bar{X}}$  = population standard deviation of the subgroup averages

The population standard deviation  $\sigma$  is needed to calculate the upper and lower control limits. Since control charts are based on sample data, Dr. Shewhart developed a good approximation of  $3\sigma_{\bar{X}}$  using the product of an  $A_2$  factor multiplied by  $\bar{R}$ , the average of the ranges. The  $A_2\bar{R}$  combination uses the sample data for its calculation.  $\bar{R}$  is calculated by summing the values of the individual subgroup ranges and dividing by the number of subgroups  $m$ :

$$\bar{R} = \frac{\sum_{i=1}^m R_i}{m}$$

where

$\bar{R}$  = average of the ranges

$R_i$  = individual range values for the sample

$m$  = number of subgroups

$A_2$ , the factor that allows the approximation  $A_2\bar{R} \approx 3\sigma_{\bar{X}}$  to be true, is selected based on the subgroup sample size  $n$ . See the appendix for the  $A_2$  factors.