## Pearson New International Edition

Structures

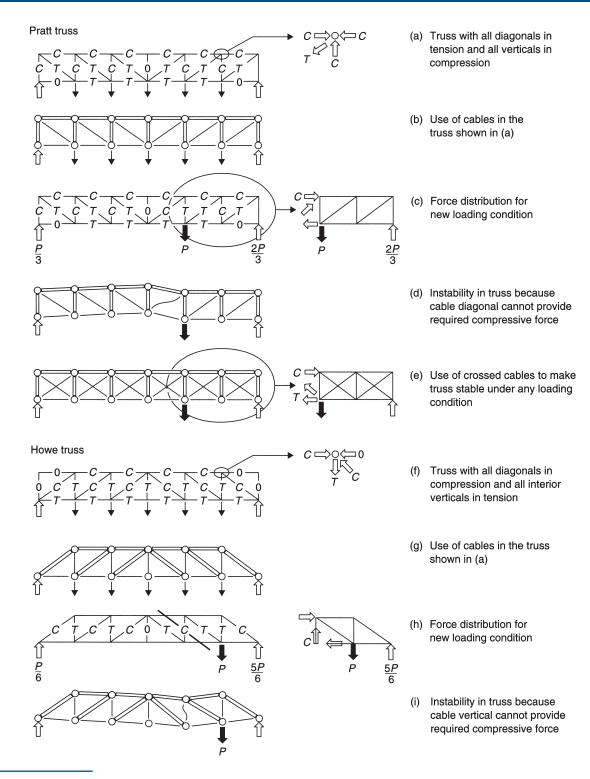
Daniel Schodek Martin Bechthold Seventh Edition



## **Pearson New International Edition**

## **Structures**

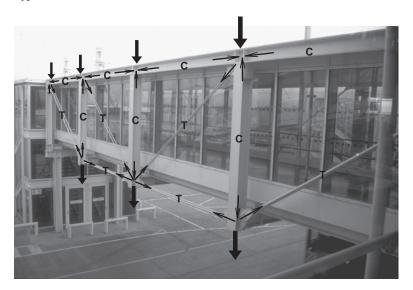
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**FIGURE 25** Parallel chord trusses: force distributions, use of cables, and effects of changing load conditions.

Whether cables can be used for tension elements in these trusses depends largely on whether the loading condition on the truss is invariant. In the diagrams shown, it is obviously suggested that cables could be suitable for use as tension elements. If members of this type are used, extreme care should be taken to ensure that these tensile members are never expected to carry compressive forces. As previously noted, stress reversals of this type could cause the truss to become unstable. Stress reversals are usually associated with a change in the pattern of external loads carried

**FIGURE 26** Parallel chord truss with cables for the diagonal members. Typical force characteristics are shown.

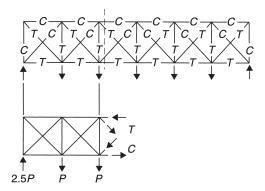


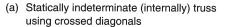
by the truss. Consider what would happen to the same two trusses under discussion if they were expected to carry the single load shown, a case that might occur if the other loads were *simply removed*. Analysis of the member forces due to these loads results in the distributions shown. Clearly, some of the members previously in tension are now in compression. If a cable were used in members previously in tension, but now in compression, the trusses would become unstable and collapse as indicated. This illustrates a fundamental design consideration: Each element in a structure must be designed to carry the forces that might develop in it under *any* possible loading condition. Satisfying this requirement could involve a large number of different load combinations.

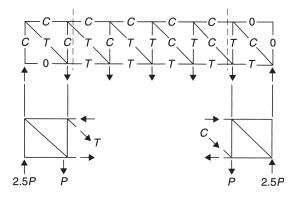
The trusses shown can be made stable for the new loading condition by using members that are capable of carrying the compressive forces indicated, rather than using cables. Alternatively, the addition of crossed cables would cause the truss shown in Figure 25(b) to be stable. The reader should carefully inspect the truss shown in Figure 25(g) to see if the addition of a crossed cable leads to stability. Clearly, it does not. For this reason, trusses having this bar pattern rarely use cables, or they are used only when the nature of the external loads are so predictable that stress reversals need not be feared. This is sometimes the case when the dead load carried by a truss significantly exceeds the live load or when there is no live load at all, because the former is usually fixed and determinable while live loads are not.

It is useful to highlight briefly two other general types of bar configuration often encountered in a constant-depth truss. The first is an internally statically indeterminate configuration, shown in Figure 27(a). An approximate force distribution for the loading indicated, based on the assumption that the diagonals are of similar stiffnesses and share equally in carrying the shear forces that are present, is also shown. The reader should study this truss closely and determine whether it is indeed stable under loading conditions other than the one illustrated. (It is, but being able to explain *why* is of crucial importance in gaining a thorough understanding of the structural behavior of trusses.) A second typical type of parallel chord truss has all the diagonals oriented in one direction [Figure 27(b)]. From a study of the previous diagonal organizations, it can be seen that in such cases diagonals on one-half of the structure are in compression and those in the other half are in tension. It makes little sense to use cables for diagonals in such a situation.

**FIGURE 27** Force distributions in parallel chord trusses using rigid diagonals that are capable of carrying either tension or compression forces.







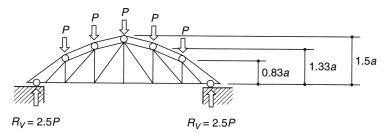
(b) Statically determinate truss with similarly sloped diagonals (Note that force states vary.)

Funicularly Shaped Trusses. Shaped trusses of the type shown at the right in Figure 24 are often quite efficient in a structural sense. These trusses are funicular shaped for the loading, reflecting the shape assumed by a flexible cable under the same loading (albeit inversely). The configuration responds to the magnitudes of the shears and moments that are present. As the bending moment varies from a maximum at the center to a minimum at the ends, the depth of the structure varies as well. (See Figure 28.) Nodes are consequently at varying heights and connecting members are sloped. This shaping leads to forces of more or less similar magnitude being developed in upper and lower chord members. (Forces in upper members are slightly higher, and not quite constant, due to their sloping.) Interestingly, if the structure is shaped precisely, the sloped chord members may also carry the shear forces involved via their vertical components. Note that the slope of the chord members increases toward the ends of the structure, where the shear forces are highest. Vertical resisting components in sloped members are thus highest as well at these points. Interstitial members carry only small forces. In fact, if shaped precisely, all interior members turn out to be zero-force members, no matter what specific internal triangulation pattern is used.

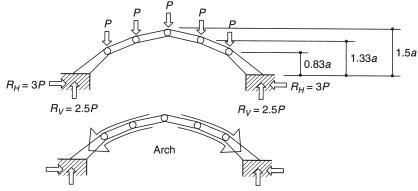
Figure 28 illustrates variations on the truss discussed. Conceptually, the zeroforce members could simply be removed to form a nontriangulated configuration without changing the ability of the structure to carry design loads to the ground. Pinned structures without diagonals are, however, of dubious practicality, because they are stable *only* under the exact loading patterns shown. If the loading pattern were changed in any way (e.g., if even one of the loads were removed or its magnitude altered), the configurations would no longer be stable and would collapse. This is because the required shear and moment resistance at the different locations is now no longer the same as that for which the structure was originally designed and the structure cannot intrinsically satisfy these new requirements as it did for the design loading. The same overall configuration with diagonals, however, is stable under not only the design loading, but any other loading condition. A function of the interstitial members is to carry forces generated by other nonuniform loads the structure must occasionally bear. If the loads were suspended from the lower chord panel points instead of being applied to the upper chord, the vertical interior members would serve as suspension rods transferring the applied loads to the upper chord members.

One final transformation of the truss shown in Figure 28(a) is of interest. The *lens*-shaped structure shown in Figure 28(c) has the same structural depth at all sections as the original structure. Its outer configuration is clearly part of the

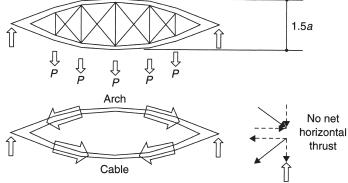
FIGURE 28 Funicular trusses: transformations of a funicular truss into related forms.



(a) Funicularly shaped truss for a series of identical and equally spaced concentrated loads. Interstitial members are zero-force members under the primary loading condition shown but carry forces when the loading changes. This is a common bridge structure.



(b) Transformation of the truss shown in (a) in which the non-load-carrying interstitial members are eliminated and the function of the lower chord (which acts as a tension tie) is replaced by buttresses. An arch structure results. If the linear segments of the structure shown are pin-connected, the assembly is stable only under the loading condition shown. Joints are thus often made rigid.



(c) A further transformation of the assemblies shown in (a) and (b) into a lens-shaped structure. The depth of the structure at any section is the same as before. One way of conceiving the structural action of the assembly is to think of it as a combination arch and cable in which the horizontal thrusts of the arch are balanced by those of the cable. Hence, no net horizontal forces develop at the foundation, only vertical forces. The structure is stable only under the loading condition shown.

same family of shapes as that present in the original truss. An analysis of this truss would reveal that the interstitial diagonals are zero-force members and thus serve only the function of stabilizing the assembly under variant loading conditions. The verticals transfer loads such that the upper and lower chord members are similarly loaded (a condition that must be met for the similarity of shape to be correct).

Instead of conceiving of this lens-shaped structure as a special form of truss, it could equally well be conceived of as a combination *arch-cable* structure. In this conception, the outward thrusts of the upper arch are balanced by the inward

**FIGURE 29** Lenticular truss: the Smithfield Stress Bridge across the Monongahela River in Pittsburgh, Pennsylvania (circa 1883).



pulls of the cable, with the consequence that there is no net lateral force present at the reaction, a result long acknowledged to be advantageous in foundation design.

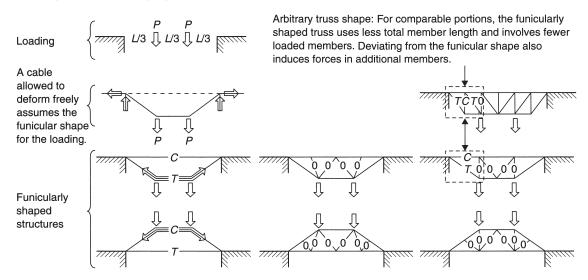
The lens-shaped form was commonly applied to many *lenticular* trusses built in the last century for use as bridges. (See Figure 29.) Additional diagonal bracing was typically used to allow the structures to carry nonuniform loading patterns.

Funicular-shaped trusses are interesting. Obviously, what is happening in these trusses is that the specific members organized along the lines of the funicular shape for the loading involved have become the dominant (actually, the only) mechanism for transferring the external loads to the supports. Other members are zero-force members serving a bracing function at most. A remarkably simple load-carrying action is thus displayed by these trusses.

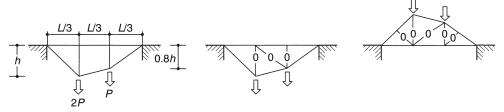
In order for the rather remarkable load-carrying actions described to occur, it is necessary that the basic shape of the structure correspond exactly to the funicular shape. Relative heights of the interior points on the trusses shown in Figures 28, 29, and 30, for example, must be determined exactly. The relative heights are clearly a function of the loads and their locations. In general, the external moments present at different sections are calculated first. A maximum depth  $d_{\rm max}$  is selected, and corresponding maximum forces in chords are calculated:  $T_{\rm max} = C_{\rm max} = M_{\rm max}/d_{\rm max}$ . The depth  $d_x$  of the structure at other sections is then varied directly in response to the moment  $M_x$ , on the assumption that the horizontal components of the chord forces are maintained constant:  $d_x = M_x/T_{\rm max} = M_x/C_{\rm max}$ . By knowing the different structural depths, member slopes can be found and final resultant forces in members calculated on the basis of known slopes and horizontal components. The vertical components of these forces will be found to balance the external shear forces present at the sections.

Because different loading conditions lead to different overall configurations (as can be appreciated from the deflected string method of arriving at funicular

**FIGURE 30** Funicular trusses: Trusses based on the funicular shape for the applied loading demonstrate a uniquely simple load-carrying action.



(a) Funicularly shaped trusses carry loads to supports in a very direct way. No forces are generated in internal members. Only those aligned with the funicular shape carry loads. If the loading changed from that shown, the shapes illustrated would no longer be funicular and forces would be induced in more members.



(b) There is only one general funicular shape for a given loading. When designing a funicularly shaped truss, care must be taken to assure that the slopes of members correspond exactly to the funicular shape (see Chapter 5).

shapes), a shape that is funicular for one loading will not be funicular for any other. This implies that forces are developed in interior members of trusses designed to be funicular for one loading when nondesign loads are present. The load-carrying action would again be more complex in such a case.

Comparing a truss not having an overall funicular shape for the loading involved with a funicular-shaped one (see Figure 30) reveals that deviating from the funicular shape not only serves to make the total member length longer, but subjects more members to primary forces (so that they would have to be designed as load-carrying, rather than zero-force, members). The consequence is that a strategy of using a funicular shape for the overall envelope of a truss may be more likely to lead to a lighter weight or minimum-volume solution than using a nonfunicular shape would.

**Other Special Shapes.** Other truss configurations that have been specially shaped to carry loads in basically a funicular way are illustrated in Figure 23. Under primary design loadings, the diagonals in these forms are zero-force members, but they are obviously needed for stability under varying loading conditions if joints are pin connected. Some actual structures reflecting such forms have been made without the aid of diagonals by using rigid, rather than pinned, connections and can carry off-balanced loads via bending in members. (Strictly speaking, these then become *rigid-frame* structures rather than trusses.)