Pearson New International Edition

# College Algebra

Robert F. Blitzer

Sixth Edition

ALWAYS LEARNING

PEARSON

# Pearson New International Edition

College Algebra
Robert F. Blitzer
Sixth Edition

PEARSON®

### General Form of the Equation of a Line

Every line has an equation that can be written in the **general form**

$$Ax + By + C = 0,$$

where $A$, $B$, and $C$ are real numbers, and $A$ and $B$ are not both zero.

**GREAT QUESTION!**

**In the general form $Ax + By + C = 0$, can I immediately determine that the slope is $A$ and the $y$-intercept is $B$?**

No. Avoid this common error. You need to solve $Ax + By + C = 0$ for $y$ before finding the slope and the $y$-intercept.

If the equation of a nonvertical line is given in general form, it is possible to find the slope, $m$, and the $y$-intercept, $b$, for the line. We solve the equation for $y$, transforming it into the slope-intercept form $y = mx + b$. In this form, the coefficient of $x$ is the slope of the line and the constant term is its $y$-intercept.

## EXAMPLE 7 Finding the Slope and the $y$-Intercept

Find the slope and the $y$-intercept of the line whose equation is $3x + 2y - 4 = 0$.

### SOLUTION

The equation is given in general form. We begin by rewriting it in the form $y = mx + b$. We need to solve for $y$.

Our goal is to isolate $y$.

$$3x + 2y - 4 = 0$$ This is the given equation.

$$2y = -3x + 4$$ Isolate the term containing $y$ by adding $-3x + 4$ to both sides.

$$\frac{2y}{2} = \frac{-3x + 4}{2}$$ Divide both sides by 2.

$$y = -\frac{3}{2}x + 2$$ On the right, divide each term in the numerator by 2 to obtain slope-intercept form.

slope: $-\frac{3}{2}$; $y$-intercept: $2$

The coefficient of $x$, $-\frac{3}{2}$, is the slope and the constant term, 2, is the $y$-intercept. This is the form of the equation that we graphed in **Figure 30.** ● ● ●

**Check Point 7** Find the slope and the $y$-intercept of the line whose equation is $3x + 6y - 12 = 0$. Then use the $y$-intercept and the slope to graph the equation.

6 Use intercepts to graph the general form of a line's equation.

## Using Intercepts to Graph $Ax + By + C = 0$

Example 7 and Check Point 7 illustrate that one way to graph the general form of a line's equation is to convert to slope-intercept form, $y = mx + b$. Then use the slope and the $y$-intercept to obtain the graph.

A second method for graphing $Ax + By + C = 0$ uses intercepts. This method does not require rewriting the general form in a different form.

### Using Intercepts to Graph $Ax + By + C = 0$

1. Find the $x$-intercept. Let $y = 0$ and solve for $x$. Plot the point containing the $x$-intercept on the $x$-axis.
2. Find the $y$-intercept. Let $x = 0$ and solve for $y$. Plot the point containing the $y$-intercept on the $y$-axis.
3. Use a straightedge to draw a line through the two points containing the intercepts. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

## EXAMPLE 8 Using Intercepts to Graph a Linear Equation

Graph using intercepts: $4x - 3y - 6 = 0$.

### SOLUTION

**Step 1 Find the $x$-intercept. Let $y = 0$ and solve for $x$.**

$$4x - 3 \cdot 0 - 6 = 0 \quad \text{Replace } y \text{ with 0 in } 4x - 3y - 6 = 0.$$
$$4x - 6 = 0 \quad \text{Simplify.}$$
$$4x = 6 \quad \text{Add 6 to both sides.}$$
$$x = \frac{6}{4} = \frac{3}{2} \quad \text{Divide both sides by 4.}$$

The $x$-intercept is $\frac{3}{2}$, so the line passes through $\left(\frac{3}{2}, 0\right)$ or $(1.5, 0)$, as shown in **Figure 33**.

**Step 2 Find the $y$-intercept. Let $x = 0$ and solve for $y$.**

$$4 \cdot 0 - 3y - 6 = 0 \quad \text{Replace } x \text{ with 0 in } 4x - 3y - 6 = 0.$$
$$-3y - 6 = 0 \quad \text{Simplify.}$$
$$-3y = 6 \quad \text{Add 6 to both sides.}$$
$$y = -2 \quad \text{Divide both sides by } -3.$$

The $y$-intercept is $-2$, so the line passes through $(0, -2)$, as shown in **Figure 33**.

**Step 3 Graph the equation by drawing a line through the two points containing the intercepts.** The graph of $4x - 3y - 6 = 0$ is shown in **Figure 33**.

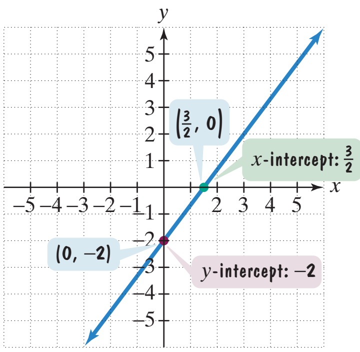



**FIGURE 33** The graph of $4x - 3y - 6 = 0$

**Check Point 8** Graph using intercepts: $3x - 2y - 6 = 0$.

We've covered a lot of territory. Let's take a moment to summarize the various forms for equations of lines.

### Equations of Lines

| | |
|---|---|
| **1.** Point-slope form | $y - y_1 = m(x - x_1)$ |
| **2.** Slope-intercept form | $y = mx + b$ or $f(x) = mx + b$ |
| **3.** Horizontal line | $y = b$ |
| **4.** Vertical line | $x = a$ |
| **5.** General form | $Ax + By + C = 0$ |

**7** Model data with linear functions and make predictions.

## Applications

Linear functions are useful for modeling data that fall on or near a line.

## EXAMPLE 9 Modeling Global Warming

The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles traps heat and raises the planet's temperature. The bar graph in **Figure 34(a)** at the top of the next page gives the average atmospheric concentration of carbon dioxide and the average global temperature for six selected years. The data are displayed as a set of six points in a rectangular coordinate system in **Figure 34(b)**.

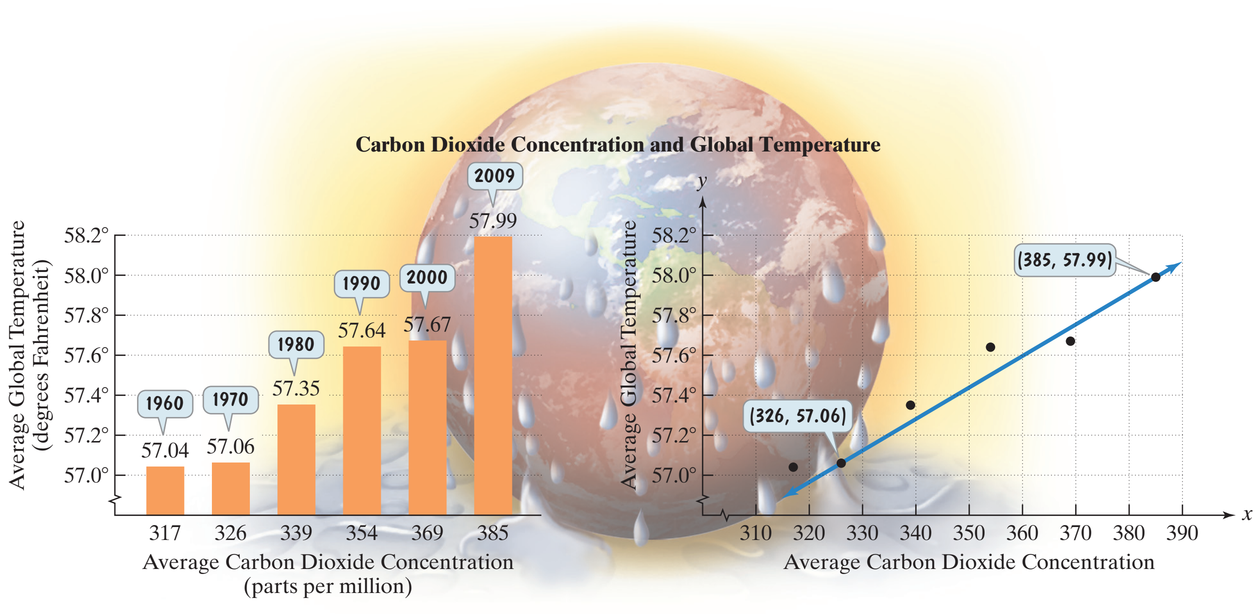



**FIGURE 34(a)**
Source: National Oceanic and Atmospheric Administration

**FIGURE 34(b)**

**a.** Shown on the scatter plot in **Figure 34(b)** is a line that passes through or near the six points. Write the slope-intercept form of this equation using function notation.

**b.** The preindustrial concentration of atmospheric carbon dioxide was 280 parts per million. The United Nations' Intergovernmental Panel on Climate Change predicts global temperatures will rise between 2°F and 5°F if carbon dioxide concentration doubles from the preindustrial level. Compared to the average global temperature of 57.99°F for 2009, how well does the function from part (a) model this prediction?

## SOLUTION

**a.** The line in **Figure 34(b)** passes through (326, 57.06) and (385, 57.99). We start by finding its slope.

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{57.99 - 57.06}{385 - 326} = \frac{0.93}{59} \approx 0.02$$

The slope indicates that for each increase of one part per million in carbon dioxide concentration, the average global temperature is increasing by approximately 0.02°F.

Now we write the line's equation in slope-intercept form.

$$y - y_1 = m(x - x_1)$$ Begin with the point-slope form.

$$y - 57.06 = 0.02(x - 326)$$ Either ordered pair can be $(x_1, y_1)$. Let $(x_1, y_1) = (326, 57.06)$. From above, $m \approx 0.02$.

$$y - 57.06 = 0.02x - 6.52$$ Apply the distributive property: $0.02(326) = 6.52$.

$$y = 0.02x + 50.54$$ Add 57.06 to both sides and solve for $y$.

A linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of $x$ parts per million is

$$f(x) = 0.02x + 50.54.$$

**b.** If carbon dioxide concentration doubles from its preindustrial level of 280 parts per million, which many experts deem very likely, the concentration will reach $280 \times 2$, or 560 parts per million. We use the linear function to predict average global temperature at this concentration.

$$f(x) = 0.02x + 50.54 \quad \text{Use the function from part (a).}$$

$$f(560) = 0.02(560) + 50.54 \quad \text{Substitute 560 for x.}$$

$$= 11.2 + 50.54 = 61.74$$

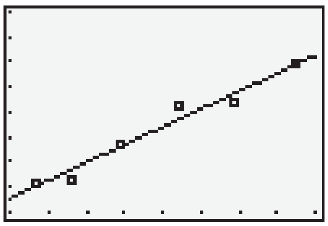
Our model projects an average global temperature of 61.74°F for a carbon dioxide concentration of 560 parts per million. Compared to the average global temperature of 57.99° for 2009 shown in **Figure 34(a)** on the previous page, this is an increase of

$$61.74°\text{F} - 57.99°\text{F} = 3.75°\text{F}.$$

This is consistent with a rise between 2°F and 5°F as predicted by the Intergovernmental Panel on Climate Change. • • •

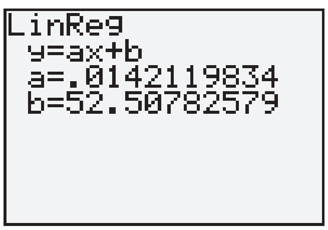
## TECHNOLOGY

You can use a graphing utility to obtain a model for a scatter plot in which the data points fall on or near a straight line. After entering the data in **Figure 34(a)** on the previous page, a graphing utility displays a scatter plot of the data and the regression line, that is, the line that best fits the data.



[310, 390, 10] by [56.8, 58.4, 0.2]

Also displayed is the regression line's equation.




**Check Point 9** Use the data points (317, 57.04) and (354, 57.64), shown, but not labeled, in **Figure 34(b)** on the previous page to obtain a linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of $x$ parts per million. Round $m$ to three decimal places and $b$ to one decimal place. Then use the function to project average global temperature at a concentration of 600 parts per million.

## CONCEPT AND VOCABULARY CHECK

*Fill in each blank so that the resulting statement is true.*

1. Data presented in a visual form as a set of points is called a/an ____________. A line that best fits this set of points is called a/an ____________ line.
2. The slope, $m$, of a line through the distinct points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula $m =$ ________.
3. If a line rises from left to right, the line has ________ slope.
4. If a line falls from left to right, the line has ________ slope.
5. The slope of a horizontal line is ________.
6. The slope of a vertical line is ____________.
7. The point-slope form of the equation of a nonvertical line with slope $m$ that passes through the point $(x_1, y_1)$ is ______________.
8. The slope-intercept form of the equation of a line is ________, where $m$ represents the ________ and $b$ represents the ________.
9. In order to graph the line whose equation is $y = \frac{2}{5}x + 3$, begin by plotting the point ________. From this point, we move ______ units up (the rise) and ______ units to the right (the run).
10. The graph of the equation $y = 3$ is a/an __________ line.
11. The graph of the equation $x = -2$ is a/an __________ line.
12. The equation $Ax + By + C = 0$, where $A$ and $B$ are not both zero, is called the ________ form of the equation of a line.

## EXERCISE SET 3

### Practice Exercises

*In Exercises 1–10, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.*

**1.** $(4, 7)$ and $(8, 10)$
**2.** $(2, 1)$ and $(3, 4)$
**3.** $(-2, 1)$ and $(2, 2)$
**4.** $(-1, 3)$ and $(2, 4)$
**5.** $(4, -2)$ and $(3, -2)$
**6.** $(4, -1)$ and $(3, -1)$
**7.** $(-2, 4)$ and $(-1, -1)$
**8.** $(6, -4)$ and $(4, -2)$
**9.** $(5, 3)$ and $(5, -2)$
**10.** $(3, -4)$ and $(3, 5)$

*In Exercises 11–38, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.*

**11.** Slope $= 2$, passing through $(3, 5)$
**12.** Slope $= 4$, passing through $(1, 3)$
**13.** Slope $= 6$, passing through $(-2, 5)$
**14.** Slope $= 8$, passing through $(4, -1)$
**15.** Slope $= -3$, passing through $(-2, -3)$
**16.** Slope $= -5$, passing through $(-4, -2)$
**17.** Slope $= -4$, passing through $(-4, 0)$
**18.** Slope $= -2$, passing through $(0, -3)$
**19.** Slope $= -1$, passing through $\left(-\frac{1}{2}, -2\right)$
**20.** Slope $= -1$, passing through $\left(-4, -\frac{1}{4}\right)$
**21.** Slope $= \frac{1}{2}$, passing through the origin
**22.** Slope $= \frac{1}{3}$, passing through the origin
**23.** Slope $= -\frac{2}{3}$, passing through $(6, -2)$
**24.** Slope $= -\frac{3}{5}$, passing through $(10, -4)$
**25.** Passing through $(1, 2)$ and $(5, 10)$
**26.** Passing through $(3, 5)$ and $(8, 15)$
**27.** Passing through $(-3, 0)$ and $(0, 3)$
**28.** Passing through $(-2, 0)$ and $(0, 2)$
**29.** Passing through $(-3, -1)$ and $(2, 4)$
**30.** Passing through $(-2, -4)$ and $(1, -1)$
**31.** Passing through $(-3, -2)$ and $(3, 6)$
**32.** Passing through $(-3, 6)$ and $(3, -2)$
**33.** Passing through $(-3, -1)$ and $(4, -1)$
**34.** Passing through $(-2, -5)$ and $(6, -5)$
**35.** Passing through $(2, 4)$ with $x$-intercept $= -2$
**36.** Passing through $(1, -3)$ with $x$-intercept $= -1$
**37.** $x$-intercept $= -\frac{1}{2}$ and $y$-intercept $= 4$
**38.** $x$-intercept $= 4$ and $y$-intercept $= -2$

*In Exercises 39–48, give the slope and y-intercept of each line whose equation is given. Then graph the linear function.*

**39.** $y = 2x + 1$
**40.** $y = 3x + 2$
**41.** $f(x) = -2x + 1$
**42.** $f(x) = -3x + 2$
**43.** $f(x) = \dfrac{3}{4}x - 2$
**44.** $f(x) = \dfrac{3}{4}x - 3$
**45.** $y = -\dfrac{3}{5}x + 7$
**46.** $y = -\dfrac{2}{5}x + 6$
**47.** $g(x) = -\dfrac{1}{2}x$
**48.** $g(x) = -\dfrac{1}{3}x$

*In Exercises 49–58, graph each equation in a rectangular coordinate system.*

**49.** $y = -2$
**50.** $y = 4$
**51.** $x = -3$
**52.** $x = 5$
**53.** $y = 0$
**54.** $x = 0$
**55.** $f(x) = 1$
**56.** $f(x) = 3$
**57.** $3x - 18 = 0$
**58.** $3x + 12 = 0$

*In Exercises 59–66,*

*a. Rewrite the given equation in slope-intercept form.*
*b. Give the slope and y-intercept.*
*c. Use the slope and y-intercept to graph the linear function.*

**59.** $3x + y - 5 = 0$
**60.** $4x + y - 6 = 0$
**61.** $2x + 3y - 18 = 0$
**62.** $4x + 6y + 12 = 0$
**63.** $8x - 4y - 12 = 0$
**64.** $6x - 5y - 20 = 0$
**65.** $3y - 9 = 0$
**66.** $4y + 28 = 0$

*In Exercises 67–72, use intercepts to graph each equation.*

**67.** $6x - 2y - 12 = 0$
**68.** $6x - 9y - 18 = 0$
**69.** $2x + 3y + 6 = 0$
**70.** $3x + 5y + 15 = 0$
**71.** $8x - 2y + 12 = 0$
**72.** $6x - 3y + 15 = 0$

### Practice Plus

*In Exercises 73–76, find the slope of the line passing through each pair of points or state that the slope is undefined. Assume that all variables represent positive real numbers. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.*

**73.** $(0, a)$ and $(b, 0)$
**74.** $(-a, 0)$ and $(0, -b)$
**75.** $(a, b)$ and $(a, b + c)$
**76.** $(a - b, c)$ and $(a, a + c)$

*In Exercises 77–78, give the slope and y-intercept of each line whose equation is given. Assume that $B \neq 0$.*

**77.** $Ax + By = C$
**78.** $Ax = By - C$

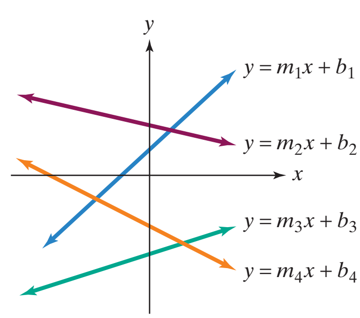
*In Exercises 79–80, find the value of y if the line through the two given points is to have the indicated slope.*

**79.** $(3, y)$ and $(1, 4)$, $m = -3$
**80.** $(-2, y)$ and $(4, -4)$, $m = \frac{1}{3}$

*In Exercises 81–82, graph each linear function.*

**81.** $3x - 4f(x) - 6 = 0$
**82.** $6x - 5f(x) - 20 = 0$
**83.** If one point on a line is $(3, -1)$ and the line's slope is $-2$, find the $y$-intercept.
**84.** If one point on a line is $(2, -6)$ and the line's slope is $-\frac{3}{2}$, find the $y$-intercept.

*Use the figure to make the lists in Exercises 85–86.*

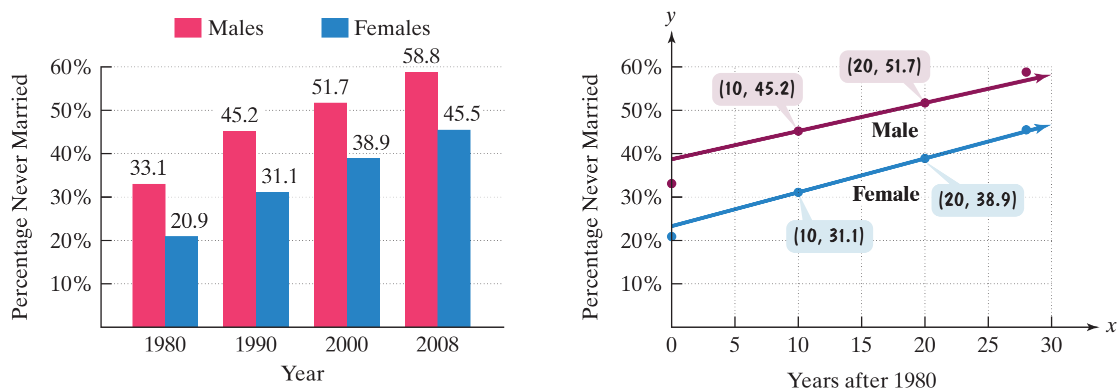



**85.** List the slopes $m_1, m_2, m_3$, and $m_4$ in order of decreasing size.
**86.** List the $y$-intercepts $b_1, b_2, b_3$, and $b_4$ in order of decreasing size.

## Application Exercises

*Americans are getting married later in life or not getting married at all. In 2008, nearly half of Americans ages 25 through 29 were unmarried. The following bar graph shows the percentage of never-married men and women in this age group. The data are displayed as two sets of four points each, one scatter plot for the percentage of never-married American men and one for the percentage of never-married American women. Also shown for each scatter plot is a line that passes through or near the four points. Use these lines to solve Exercises 87–88.*

**Percentage of United States Population Never Married, Ages 25–29**




Source: U.S. Census Bureau

**87.** In this exercise, you will use the blue line for the women shown on the scatter plot to develop a model for the percentage of never-married American females ages 25–29.

**a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American females ages 25–29, $y$, $x$ years after 1980.

**b.** Write the equation from part (a) in slope-intercept form. Use function notation.

**c.** Use the linear function to predict the percentage of never-married American females, ages 25–29, in 2020.

**88.** In this exercise, you will use the red line for the men shown on the scatter plot to develop a model for the percentage of never-married American males ages 25–29.

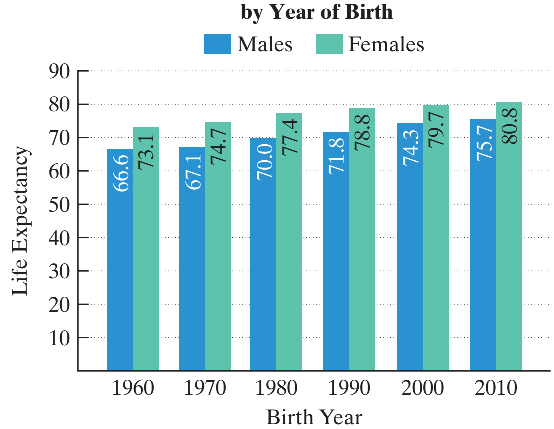
**a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American males ages 25–29, $y$, $x$ years after 1980.

**b.** Write the equation from part (a) in slope-intercept form. Use function notation.

**c.** Use the linear function to predict the percentage of never-married American males, ages 25–29, in 2015.

*The bar graph gives the life expectancy for American men and women born in six selected years. In Exercises 89–90, you will use the data to obtain models for life expectancy and make predictions about how long American men and women will live in the future.*

**Life Expectancy in the United States, by Year of Birth**




Source: National Center for Health Statistics

**89.** Use the data for males shown in the bar graph at the bottom of the previous column to solve this exercise.

**a.** Let $x$ represent the number of birth years after 1960 and let $y$ represent male life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.

**b.** Draw a line through the two points that show male life expectancies for 1980 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American men born $x$ years after 1960.

**c.** Use the function from part (b) to project the life expectancy of American men born in 2020.

**90.** Use the data for females shown in the bar graph at the bottom of the previous column to solve this exercise.

**a.** Let $x$ represent the number of birth years after 1960 and let $y$ represent female life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.

**b.** Draw a line through the two points that show female life expectancies for 1970 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American women born $x$ years after 1960. Round the slope to two decimal places.

**c.** Use the function from part (b) to project the life expectancy of American women born in 2020.