Pearson New International Edition

Systems Engineering and Analysis

Blanchard Fabrycky

Fifth Edition

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$460 per unit. It is estimated that the additional fixed cost would be $80,000 per year if the component is rebuilt. Find the number of units per year for which the cost of the two alternatives will break even.

20. A marketing company can lease a fleet of automobiles for its sales personnel for $35 per day plus $0.18 per mile for each vehicle. As an alternative, the company can pay each salesperson $0.45 per mile to use his or her own automobile. If these are the only costs to the company, how many miles per day must a salesperson drive for the two alternatives to break even?

21. An electronics manufacturer is considering the purchase of one of two types of laser trimming devices. The sales forecast indicated that at least 8,000 units will be sold per year. Device A will increase the annual fixed cost of the plant by $20,000 and will reduce variable cost by $5.60 per unit. Device B will increase the annual fixed cost by $5,000 and will reduce variable cost by $3.60 per unit. If variable costs are now $20 per unit produced, which device should be purchased?

22. Machine A costs $20,000, has zero salvage value at any time, and has an associated labor cost of $1.15 for each piece produced on it. Machine B costs $36,000, has zero salvage value at any time, and has an associated labor cost of $0.90. Neither machine can be used except to produce the product described. If the interest rate is 10% and the annual rate of production is 20,000 units, how many years will it take for the cost of the two machines to break even?

23. An electronics manufacturer is considering two methods for producing a circuit board. The board can be hand-wired at an estimated cost of $9.80 per unit and an annual fixed equipment cost of $10,000. A printed equivalent can be produced using equipment costing $180,000 with a service life of 8 years and salvage value of $12,000. It is estimated that the labor cost will be $3.20per unit and that the processing equipment will cost $4,000 per year to maintain. If the interest rate is 8%, how many circuit boards must be produced each year for the two methods to break even?

24. It is estimated that the annual sales of an energy saving device will be 20,000 the first year and increase by 10,000 per year until 50,000 units are sold during the fourth year. Proposal A is to purchase manufacturing equipment costing $120,000 with an estimated salvage value of $15,000 at the end of 4 years. Proposal B is to purchase equipment costing $280,000 with an estimated salvage value of $32,000 at the end of 4 years. The variable manufacturing cost per unit under proposal A is estimated to be $8.00, but is estimated to be only $2.60 under proposal B. If the interest rate is 9%, which proposal should be accepted for a 4-year production horizon?

25. The fixed operating cost of a machine center (capital recovery, interest, maintenance, space charges, supervision, insurance, and taxes) is F dollars per year. The variable cost of operating the center (power, supplies, and other items, but excluding direct labor) is V dollars per hour of operation. If N is the number of hours the center is operated per year, TC the annual total cost of operating the center, TC_h the hourly cost of operating the center, t the time in hours to process 1 unit of product, and M the center cost of processing 1 unit, write expressions for (a) TC, (b) TC_h, and (c) M.

26. In Problem 25, $F = \$60{,}000$ per year, $t = 0.2$ hour, $V = \$50$ per hour, and N varies from 1,000 to 10,000 in increments of 1,000.

a. Plot values of M as a function of N.

b. Write an expression for the total cost of direct labor and machine cost per unit, TC_u, using the symbols in Problem 25 and letting W equal the hourly cost of direct labor.

27. A certain firm has the capacity to produce 800,000 units per year. At present, it is operating at 75% of capacity. The income per unit is $0.10 regardless of the output. Annual fixed costs are $28,000, and the variable cost is $0.06 per unit. Find the annual profit or loss at this capacity and the capacity for which the firm will break even.

28. An arc welding machine that is used for a certain joining process costs $90,000. The machine has a life of 5 years and a salvage value of $10,000. Maintenance, taxes, insurance, and other fixed costs amount to $5,000 per year. The cost of power and supplies is $28.00 per hour of operation and the total operator cost (direct and indirect) is $65.00 per hour. If the cycle time per unit of product is 60 minutes and the interest rate is 8%, calculate the cost per unit if (a) 200, (b) 600, and (c) 1,800 units of output are needed per year.

29. A certain processing center has the capacity to assemble 650,000 units per year. At present, it is operating at 65% of capacity. The annual income is $416,000. Annual fixed cost is $192,000 and the variable cost is $0.38 per unit assembled.

a. What is the annual profit or loss attributable to the center?
b. At what volume of output does the center break even?
c. What will be the profit or loss at 70%, 80%, and 90% of capacity on the basis of constant income per unit and constant variable cost per unit?

30. Chemco operates two plants, A and B, which produce the same product. The capacity of plant A is 60,000 gallons while that of B is 80,000 gallons. The annual fixed cost of plant A is $2,600,000 per year and the variable cost is $32 per gallon. The corresponding values for plant B are $2,800,000 and $39 per gallon. At present, plants A and B are being operated at 35% and 40% of capacity, respectively.

a. What would be the total cost of production of plants A and B?
b. What are the total cost and the average unit cost of the total output of both plants?
c. What would be the total cost to the company and cost per gallon if all production were transferred to plant A?
d. What would be the total cost to the company and cost per gallon if all production were transferred to plant B?

Optimization in Design and Operations

The design of complex systems that appropriately incorporate optimization in the design process is an important challenge facing the systems engineer. A parallel but lesser challenge arises from the task of optimizing the operation of systems already in being. In the former case, optimum values of design variables are sought for each instance of the design-dependent parameter set. In the latter, optimum values for policy variables are desired. The modeling approaches are essentially the same. This distinction is important only as a means for contrasting system design and systems analysis.

In this chapter, the general approach to the formulation and manipulation of mathematical models is presented. The specific mathematical methods used vary in degree of complexity and depend on the system under study. Examples are used to illustrate design situations for static and dynamic systems. Both unconstrained and constrained examples are presented. Classical optimization techniques are illustrated first, followed by some nonclassical approaches, including linear programming methods.

These fundamental areas are developed in detail in this chapter by the integration of economic modeling and optimization, with hypothetical examples from design and operations. Consideration and study of this material will impart a general understanding of optimization as a means to an end, not as an end in itself. Specifically, the reader should become proficient in several areas:

- Utilizing calculus-based classical optimization concepts and methods for both univariate and multivariate applications in design and operations;
- Understanding that the utilization of optimization as presented in this chapter is traceable back to and inspired by the field of operations research;
- Applying decision theory and economic evaluation methods jointly to expand and realize the broader benefit of equivalence;
- Structuring unconstrained situations requiring optimization as a prelude to addressing life-cycle-based money flow representations;

- Considering investment cost optimization as a step on the way to determining life-cycle complete evaluations of design and operations;
- Applying optimization in the face of constraints as an extension of the unconstrained situations and examples;
- Evaluating situations involving optimization and multiple criteria, employing the decision evaluation display; and
- Understanding and applying constrained optimization by both the graphical and the simplex methods of linear programming.

The final section of this chapter provides a summary of the broad domain of optimization with selected example applications in design and operations. It also provides some recommended sources of additional material useful in extending insight and improving understanding.

1 CLASSICAL OPTIMIZATION THEORY

Optimization is the process of seeking the best. In systems engineering and analysis, this process is applied to each alternative in accordance with decision evaluation theory. In so doing, the general system optimization function of Equation A.1 as well as the decision evaluation function given by Equation A.2 are exercised and illustrated by hypothetical examples. Before presenting specific derivations and examples of optimization in design and operations, this section provides a review of calculus-based methods for optimizing functions with one or more decision variables.

1.1 The Slope of a Function

The slope of a function $y = f(x)$ is defined as the rate of change of the dependent variable, y, divided by the rate of change of the independent variable, x. If a positive change in x results in a positive change in y, the slope is positive. Conversely, a positive change in x resulting in a negative change in y indicates a negative slope.

If $y = f(x)$ defines a straight line, the difference between any two points x_1 and x_2 represents the change in x, and the difference between any two points y_1 and y_2 represents the change in y. Thus, the rate of change of y with respect to x is $(y_1 - y_2)/(x_1 - x_2)$, the slope of the straight line. This slope is constant for all points on the straight line; $\Delta y/\Delta x = \text{constant}$.

For a nonlinear function, the rate of change of y with respect to changes in x is not constant but changes with changes in x. The slope must be evaluated at each point on the curve. This can be done by assuming an arbitrary point on the function p, for which the x and y values are x_0 and $f(x_0)$, as shown in Figure 1. The rate of change of y with respect to x at point p is equal to the slope of a line tangent to the function at that point. It is observed that the rate of change of y with respect to x differs from that at p at other points on the curve.

1.2 Differential Calculus Fundamentals

Differential calculus is a mathematical tool for finding successively better and better approximations for the slope of the tangent line shown in Figure 1. Consider another point on $f(x)$ designated q, situated at an x distance from point p equal to Δx and situated at a y distance from point p equal to Δy. Then the slope of a line segment through points p and q would have a slope $\frac{\Delta y}{\Delta x}$. But this is the average slope of $f(x)$

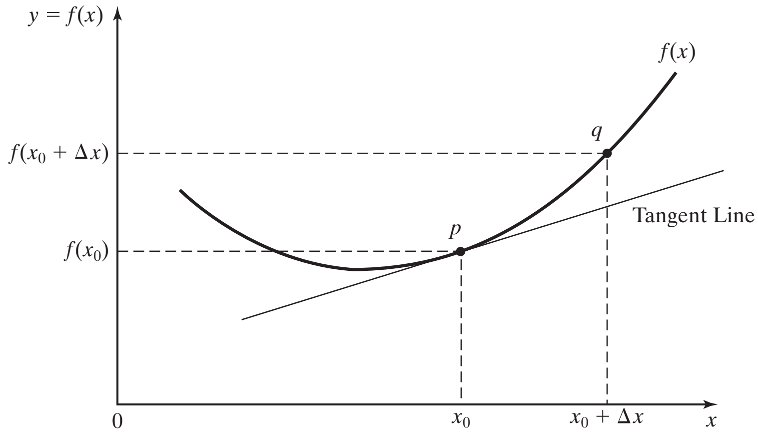

Figure 1 Slope of a function.

between the designated points. In classical optimization, it is the instantaneous rate of change at a given point that is sought.

The instantaneous rate of change in $f(x)$ at $x = x_0$ can be found by letting $\Delta x \to 0$. Referring to Figure 1, this can be stated as

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \tag{1}$$

It is noted that as Δx becomes smaller and smaller, the line segment passing through points p and q approaches the tangent line to point p. Thus, the slope of the function at x_0 is given by Equation 1 when Δx is infinitesimally small.

Equation 1 is an expression for slope of general applicability. It is called a *derivative*, and its application is a process known as *differentiation*. For the function $y = f(x)$, the symbolism dy/dx or $f'(x)$ is most often used to denote the derivative.

As an example of the process of differentiation utilizing Equation 1, consider the function $y = f(x) = 8x - x^2$. Substituting into Equation 1 gives

$$\begin{aligned}
\lim_{\Delta x \to 0} &= \frac{[8(x + \Delta x) - (x + \Delta x)^2] - [8x - x^2]}{\Delta x} \\
&= \frac{8x + 8\Delta x - x^2 - 2x\Delta x - \Delta x^2 - 8x + x^2}{\Delta x} \\
&= \frac{8\Delta x - 2x\Delta x - \Delta x^2}{\Delta x} \\
&= 8 - 2x - \Delta x
\end{aligned}$$

As $\Delta x \to 0$ the derivative, $dy/dx = 8 - 2x$, the slope at any point on the function. For example, at $x = 4$, $dy/dx = 0$, indicating that the slope of the function is zero at $x = 4$.

An alternative example of the process of differentiation, not tied to Equation 1, might be useful to consider. Begin with a function of some form such as $y = f(x) = 3x^2 + x + 2$.

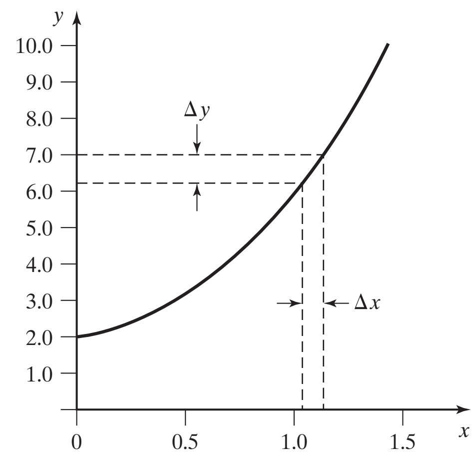

Figure 2 Function $3x^2 + x + 2$.

Let x increase by an amount Δx. Then y will increase by an amount Δy as shown in Figure 2, giving

$$\begin{aligned} y + \Delta y &= 3(x + \Delta x)^2 + (x + \Delta x) + 2 \\ &= 3[x^2 + 2x(\Delta x) + (\Delta x)^2] + (x + \Delta x) + 2 \\ \Delta y &= 3[x^2 + 2x(\Delta x) + (\Delta x)^2] + (x + \Delta x) + 2 - y \end{aligned}$$

But $y = 3x^2 + x + 2$, giving

$$\begin{aligned} \Delta y &= 3x^2 + 6x(\Delta x) + 3(\Delta x)^2 + x + \Delta x + 2 - 3x^2 - x - 2 \\ &= 6x(\Delta x) + 3(\Delta x)^2 + \Delta x \end{aligned}$$

The average rate of change of Δy with respect to Δx is

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{6x(\Delta x)}{\Delta x} + \frac{3(\Delta x)^2}{\Delta x} + \frac{\Delta x}{\Delta x} \\ &= 6x + 3(\Delta x) + 1 \end{aligned}$$

But because

$$\underset{\Delta x:\ 0}{\text{limit}} = \frac{dy}{dx} = f'(x)$$

the instantaneous rate of change is $6x + 1$.

Instead of proceeding as earlier for each case encountered, certain rules of differentiation have been derived for a range of common functional forms. Some of these are as follows:

1. Derivative of a constant: If $f(x) = k$, a constant, then $dy/dx = 0$.
2. Derivative of a variable: If $f(x) = x$, a variable, then $dy/dx = 1$.