PEARSON NEW INTERNATIONAL EDITION

# Modern Physics

Randy Harris

Second Edition

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**5.** Just what is stationary in a stationary state? The particle? Something else?

**6.** When is the temporal part of the wave function 0? Why is this important?

**7.** We say that the ground state for the particle in a box has nonzero energy. What goes wrong with $\psi$ in equation (16) if $n = 0$?

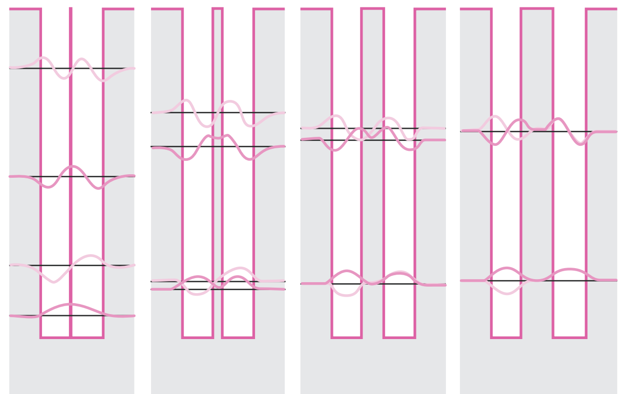
**8.** Equation (16) gives infinite well energies. Because equation (22) cannot be solved in closed form, there is no similar compact formula for finite well energies. Still, many conclusions can be drawn without one. Argue on largely qualitative grounds that if the walls of a finite well are moved closer together but not changed in height, then the well must progressively hold fewer bound states. (Make a clear connection between the width of the well and the height of the walls.)

**9.** A *half*-infinite well has an infinitely high wall at the origin and one of finite height $U_0$ at $x = L$. Like the finite well, the number of allowed states is limited. Assume that it has two states, of energy $E_1$ and $E_2$, where $E_2$ is not much below $U_0$. Make a sketch of the potential energy, then add plausible sketches of the two allowed wave functions on separate horizontal axes whose heights are $E_1$ and $E_2$.

**10.** Consider a particle in the ground state of a finite well. Describe the changes in its wave function and energy as the walls are made progressively higher ($U_0$ is increased) until essentially infinite.

**11.** A particle is subject to a potential energy that has an essentially infinitely high wall at the origin, like the infinite well, but for positive values of $x$ is of the form $U(x) = -b/x$, where $b$ is a constant. (a) Sketch this potential energy. (b) How much energy could a classical particle have and still be bound by such a potential energy? (c) Add to your sketch a plot of $E$ for a bound particle and indicate the outer classical turning point (the inner being the origin). (d) Assuming that a quantum-mechanical description is in order, sketch a plausible ground-state wave function, making sure that your function's second derivative is of the proper sign when $U(x)$ is less than $E$ and when it is greater.

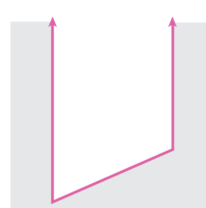
**12.** Simple models are very useful. Consider the twin finite wells shown in the figure, at first with a tiny separation, then with increasingly distant separations. In all cases, the four lowest allowed wave functions are plotted on axes proportional to their energies. We see that they pass through the classically forbidden region between the wells, and we also see a trend. When the wells are very close, the four functions and energies are what we might expect of a single finite well, but as they move apart, pairs of functions converge to intermediate energies. (a) The energies of the second and fourth states decrease. Based on changing wavelength alone, argue that this is reasonable. (b) The energies of the first and third states *increase*. Why? (*Hint:* Study how the behavior required in the classically forbidden region affects these two relative to the others.) (c) The distant-wells case might represent two distant atoms. If each atom had one electron, what advantage is there in bringing the atoms closer to form a molecule? (*Note:* Two electrons can have the same wave function.)



**13.** In the harmonic oscillator wave functions of Figure 18, there is variation in wavelength from the middle to the extremes of the classically allowed region, most noticeable in the higher-$n$ functions. Why does it vary as it does?

**14.** Summarize the similarities and differences between the three simple bound cases considered in this chapter.

**15.** Consider a particle bound in an infinite well, where the potential inside is not constant but a linearly varying function. Suppose the particle is in a fairly high energy state, so that its wave function stretches across the entire well; that is, it isn't caught in the "low spot." Decide how, if at all, its wavelength should vary. Then sketch a plausible wave function.



**16.** The quantized energy levels in the infinite well get farther apart as $n$ increases, but in the harmonic oscillator they are equally spaced. (a) Explain the difference by considering the distance "between the walls" in each case and how it depends on the particle's energy.

(b) A very important bound system, the hydrogen atom, has energy levels that actually get closer together as $n$ increases. How do you think the separation between the potential energy "walls" in this system varies relative to the other two? Explain.

**17.** In several bound systems, the quantum-mechanically allowed energies depend on a single quantum number. We found in Section 5 that the energy levels in an infinite well are given by $E_n = a_1 n^2$, where $n = 1, 2, 3, \ldots$ and $a_1$ is a constant. (Actually, we know what $a_1$ is, but it would only distract us here.) Section 7 showed that for a harmonic oscillator, they are $E_n = a_2(n - \frac{1}{2})$, where $n = 1, 2, 3, \ldots$. (Using an $n - \frac{1}{2}$ with $n$ strictly positive is equivalent to $n + \frac{1}{2}$ with $n$ nonnegative.) Finally, for a hydrogen atom, a bound system $E_n = -a_3/n^2$, where $n = 1, 2, 3, \ldots$. Consider particles making downward transitions between the quantized energy levels, each transition producing a photon. For each of these three systems, is there a minimum photon wavelength? A maximum? It might be helpful to make sketches of the relative heights of the energy levels in each case.

**18.** Quantum-mechanical stationary states are of the general form $\Psi(x, t) = \psi(x)\, e^{-i\omega t}$. For the basic plane wave, this is $\Psi(x, t) = A\, e^{ikx}\, e^{-i\omega t} = Ae^{i(kx-\omega t)}$, and for a particle in a box, it is $\Psi(x, t) = A \sin(kx)\, e^{-i\omega t}$. Although both are sinusoidal, we claim that the plane wave alone is the prototype function whose momentum is pure—a well-defined value in one direction. Reinforcing the claim is the fact that the plane wave alone lacks features that we expect to see only when, effectively, waves are moving in both directions. What features are these, and, considering the probability densities, are they indeed present for a particle in a box and absent for a plane wave?

# Exercises

## Section 3

**19.** Under what circumstance does the integral $\int_{x_0}^{\infty} x^b dx$ diverge? Use this to argue that a physically acceptable wave function must fall to 0 faster than $|x|^{-1/2}$ does as $x$ gets large.

## Section 4

**20.** A comet in an extremely elliptical orbit about a star has, of course, a maximum orbit radius. By comparison, its minimum orbit radius may be nearly 0. Make plots of the potential energy and a plausible total energy $E$ versus radius on the same set of axes. Identify the classical turning points on your plot.

**21.** A classical particle confined to the positive $x$-axis experiences a force whose potential energy is

$$U(x) = \frac{1}{x^2} - \frac{2}{x} + 1 \quad \text{(SI units)}$$

(a) By finding its minimum value and determining its behaviors at $x = 0$ and $x = +\infty$, sketch this potential energy.
(b) Suppose the particle has an energy of 0.5 J. Find any turning points. Would the particle be bound?
(c) Suppose the particle has an energy of 2.0 J. Find any turning points. Would the particle be bound?

## Section 5

**22.** A study of classical waves tells us that a standing wave can be expressed as a sum of two traveling waves. Quantum-mechanical traveling waves, are of the form $\Psi(x, t) = A\, e^{i(kx-\omega t)}$. Show that the infinite well's standing-wave function can be expressed as a sum of two traveling waves.

**23.** Write out the total wave function $\Psi(x, t)$ for an electron in the $n = 3$ state of a 10 nm wide infinite well. Other than the symbols $x$ and $t$, the function should include only numerical values.

**24.** An electron in the $n = 4$ state of a 5 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

**25.** An electron is trapped in a quantum well (practically infinite). If the lowest-energy transition is to produce a photon of 450 nm wavelength, what should be the well's width?

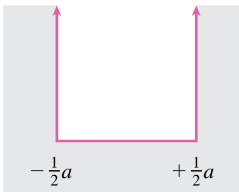
**26.** Because protons and neutrons are similar in mass, size, and certain other characteristics, a collective term, *nucleons,* has been coined that encompasses both of these constituents of the atomic nucleus. In many nuclei, nucleons are confined (by the strong force) to dimensions of roughly 15 femtometers. Photons emitted by nuclei as the nucleons drop to lower energy levels are known as gamma particles. Their energies are typically in the MeV range. Why does this make sense?

**27.** Where would a particle in the first excited state (first above ground) of an infinite well mostly likely be found?

**28.** What is the probability that a particle in the first excited ($n = 2$) state of an infinite well would be found in the middle third of the well? How does this compare with the classical expectation? Why?

**29.** A tiny 1 μg particle is in a 1 cm wide enclosure and takes a year to bounce from one end to the other and back. (a) How many nodes are there in its enclosure? (b) How would your answer change if the particle were more massive or moving faster?

* **30.** A particle is bound by a potential energy of the form




$$U(x) = \begin{cases} 0 & |x| < \frac{1}{2}a, \\ \infty & |x| > \frac{1}{2}a \end{cases}$$

This differs from the infinite well of Section 5 in being symmetric about $x = 0$, which implies that the probability densities must also be symmetric. Noting that either sine *or* cosine would fit this requirement and could be made continuous with the zero wave function outside the well, determine the allowed energies and corresponding normalized wave functions for this potential well.

## Section 6

**31.** Verify that solution (19) satisfies the Schrödinger equation in form (18).

**32.** A finite potential energy function $U(x)$ allows $\psi(x)$, the solution of the time-independent Schrödinger equation, to penetrate the classically forbidden region. Without assuming any particular function for $U(x)$, show that $\psi(x)$ must have an inflection point at any value of $x$ where it enters a classically forbidden region.

**33.** Verify that $A \sin(kx) + B \cos(kx)$ is a solution of equation (12).

**34.** A 50 eV electron is trapped between electrostatic walls 200 eV high. How far does its wave function extend beyond the walls?

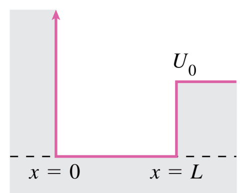
**35.** An electron is trapped in a finite well. How "far" (in eV) is it from being free if the penetration length of its wave function into the classically forbidden region is 1 nm?

**36.** Whereas an infinite well has an infinite number of bound states, a finite well does not. By relating the well height $U_0$ to the kinetic energy and the kinetic energy (through $\lambda$) to $n$ and $L$, show that the number of bound states is given roughly by $\sqrt{8mL^2U_0/h^2}$. (Assume that the number is large.)

**37.** The deeper the finite well, the more states it holds. In fact, a new state, the $n$th, is added when the well's depth $U_0$ reaches $h^2(n-1)^2/8mL^2$. (a) Argue that this should be the case based only on $k = \sqrt{2mE/\hbar^2}$, the shape of the wave inside, and the degree of penetration of the classically forbidden region expected for a state whose energy $E$ is only negligibly below $U_0$. (b) How many states would be found up to this same "height" in an *infinite* well.

**Exercises 38–41 refer to a particle of mass *m* trapped in a half-infinite well, with potential energy given by**

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ U_0 & x \ge L \end{cases}$$




**38.** Advance an argument based on $p = h/\lambda$ that there is no bound state in a half-infinite well unless $U_0$ is at least $h^2/32mL^2$. (*Hint:* What is the maximum wavelength possible within the well?)

**39.** A finite well always has at least one bound state. Why does the argument of Exercise 38 fail in the case of a finite well?

* **40.** Write solutions to the Schrödinger equation appropriate in the various regions, impose required continuity conditions, and obtain the energy quantization condition:

$$\sqrt{E}\cot\left(\frac{\sqrt{2mE}}{\hbar}L\right) = -\sqrt{U_0 - E}$$

* **41.** Using the result of Exercise 40 and a computer or calculator able to solve (transcendental) equations, find the two bound-state energies for a well in which $U_0 = 4(\pi^2\hbar^2/2mL^2)$.

* **42.** By largely qualitative arguments, Exercise 37 shows that a finite well can hold an $n$th state only if its depth $U_0$ is at least $h^2(n-1)^2/8mL^2$. Show that this result also follows from equation (23) and the accompanying Figure 14.

* **43.** Obtain expression (23) from equation (22).
Using $\cos\theta = \cos^2(\frac{1}{2}\theta) - \sin^2(\frac{1}{2}\theta)$ and $\sin\theta = 2\sin(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta)$, first convert the argument of the cotangent from $kL$ to $\frac{1}{2}kL$. Next, put the resulting equation in quadratic form, and then factor. Note that $\alpha$ is positive by definition.

* **44.** Using equation (23), find the energy of a particle confined to a finite well whose walls are half the height of the ground-state infinite well energy, $E_1$. (A calculator or computer able to solve equations numerically may be used, but this happens to be a case where an exact answer can be deduced without too much trouble.)

* **45.** There are mathematical solutions to the Schrödinger equation for the finite well for *any* energy, and in fact, they can be made smooth everywhere. Guided by A Closer Look: Solving the Finite Well, show this as follows:

(a) Don't throw out any mathematical solutions. That is, in region II ($x < 0$), assume that $\psi(x) = Ce^{+\alpha x} + De^{-\alpha x}$, and in region III ($x > L$), assume that $\psi(x) = Fe^{+\alpha x} + Ge^{-\alpha x}$. Write the smoothness conditions.

(b) In Section 6, the smoothness conditions were combined to eliminate $A$, $B$, and $G$ in favor of $C$. In the remaining equation, $C$ canceled, leaving an equation involving only $k$ and $\alpha$, solvable for only certain values of $E$. Why can't this be done here?

(c) Our solution is smooth. What is still wrong with it physically?

(d) Show that

$$D = \frac{1}{2}\left(B - \frac{k}{\alpha}A\right) \quad \text{and}$$

$$F = \frac{1}{2}e^{-\alpha L}\left[\left(A - B\frac{k}{\alpha}\right)\sin(kL) + \left(A\frac{k}{\alpha} + B\right)\cos(kL)\right]$$

and that setting these offending coefficients to 0 reproduces quantization condition (22).

* **46.** It is possible to take the finite well wave functions further than (21) without approximation, eliminating all but one normalization constant $C$. First, use the continuity/smoothness conditions to eliminate $A$, $B$, and $G$ in favor of $C$ in (21). Then make the change of variables $z \equiv x - L/2$ and use the trigonometric relations $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$ and $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ on the functions in region I, $-L/2 < z < +L/2$. The change of variables shifts the problem so that it is symmetric about $z = 0$, which requires that the probability density be symmetric and thus that $\psi(z)$ be either an odd or even function of $z$. By comparing the region II and region III functions, argue that this in turn demands that $(\alpha/k)\sin(kL) + \cos(kL)$ must be either $+1$ (even) or $-1$ (odd). Next, show that these conditions can be expressed, respectively, as $\alpha/k = \tan(kL/2)$ and $\alpha/k = -\cot(kL/2)$. Finally, plug these separately back into the region I solutions and show that

$$\psi(z) = C \times \begin{cases} e^{+\alpha(z+\frac{1}{2}L)} & z < -\frac{1}{2}L \\ \dfrac{\cos kz}{\cos(\frac{1}{2}kL)} & -\frac{1}{2}L < z < +\frac{1}{2}L \\ e^{-\alpha(z-\frac{1}{2}L)} & z > +\frac{1}{2}L \end{cases}$$

or

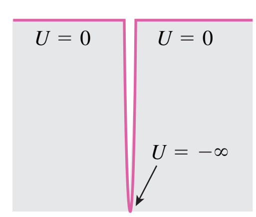
$$\psi(z) = C \times \begin{cases} e^{+\alpha(z+\frac{1}{2}L)} & z < -\frac{1}{2}L \\ \dfrac{-\sin kz}{\sin(\frac{1}{2}kL)} & -\frac{1}{2}L < z < +\frac{1}{2}L \\ -e^{-\alpha(z-\frac{1}{2}L)} & z > +\frac{1}{2}L \end{cases}$$

Note that $C$ is now a standard multiplicative normalization constant. Setting the integral of $|\psi(z)|^2$ over all space to 1 would give it in terms of $k$ and $\alpha$, but because we can't solve (22) exactly for $k$ (or $E$), neither can we obtain an exact value for $C$.

* **47.** Consider the delta well potential energy:

$$U(x) = \begin{cases} 0 & x \neq 0 \\ -\infty & x = 0 \end{cases}$$

Although not completely realistic, this potential energy is often a convenient approximation to a *very* strong, *very* narrow attractive potential energy well. It has only one allowed bound-state wave function, and because the top of the well is defined as $U = 0$, the corresponding bound-state energy is negative. Call its value $-E_0$.



(a) Applying the usual arguments and required continuity conditions (need it be smooth?), show that the wave function is given by

$$\psi(x) = \left(\frac{2mE_0}{\hbar^2}\right)^{1/4} e^{-\left(\sqrt{2mE_0}/\hbar\right)|x|}$$

(b) Sketch $\psi(x)$ and $U(x)$ on the same diagram. Does this wave function exhibit the expected behavior in the classically forbidden region?

* **48.** Figure 15 shows the allowed wave functions for a finite well whose depth $U_0$ was chosen to be $6\pi^2\hbar^2/mL^2$. (a) Insert this value in equation (23), then, using a calculator or computer, solve for the allowed values of $kL$, of which there are four. (b) Using $k = \sqrt{2mE}/\hbar$, find the corresponding values of $E$. Do they appear to agree with Figure 15? (c) Show that the chosen $U_0$ value implies that $\alpha = \sqrt{(12\pi^2/L^2) - k^2}$. (d) Defining $L$ and $C$ to be 1 for convenience, plug your $kL$ and $\alpha$ values into the wave function given in Exercise 46, then plot the results. (*Note:* Your first and third $kL$ values should correspond to even functions of $z$, thus using the form with cos $kz$, while the second and fourth correspond to odd functions.) Do the plots also agree with Figure 15?

## Section 7

**49.** For the harmonic oscillator potential energy, $U = \frac{1}{2}\kappa x^2$, the ground-state wave function is $\psi(x) = Ae^{-\left(\sqrt{m\kappa}/2\hbar\right)x^2}$, and its energy is $\frac{1}{2}\hbar\sqrt{\kappa/m}$.

(a) Find the classical turning points for a particle with this energy.
(b) The Schrödinger equation says that $\psi(x)$ and its second derivative should be of the opposite sign when $E > U$ and of the same sign when $E < U$. These two regions are divided by the classical turning points. Verify the relationship between $\psi(x)$ and its second derivative for the ground-state oscillator wave function. (*Hint:* Look for the inflection points.)

**50.** A 2 kg block oscillates with an amplitude of 10 cm on a spring of force constant 120 N/m. (a) In which quantum state is the block? (b) The block has a slight electric charge and drops to a lower energy level by generating a photon. What is the minimum energy decrease possible, and what would be the corresponding fractional change in energy?

**51.** Air is mostly $N_2$, diatomic nitrogen, with an effective spring constant of $2.3 \times 10^3$ N/m, and an effective oscillating mass of half the atomic mass. For roughly what temperatures should vibration contribute to its heat capacity?

**52.** To a good approximation, the hydrogen chloride molecule, HCl, behaves vibrationally as a quantum harmonic oscillator of spring constant 480 N/m and with effective oscillating mass just that of the lighter atom, hydrogen. If it were in its ground vibrational state, what wavelength photon would be just right to bump this molecule up to its next-higher vibrational energy state?

**53.** In Section 5, it was shown that the infinite well energies follow simply from $\lambda = h/p$; the formula for kinetic energy, $p^2/2m$; and a famous standing-wave condition, $\lambda = 2L/n$. The arguments are perfectly valid when the potential energy is 0 (inside the well) and $L$ is strictly constant, but they can also be useful in other cases. The length $L$ allowed the wave should be roughly the distance between the classical turning points, where there is no kinetic energy left. Apply these arguments to the oscillator potential energy, $U(x) = \frac{1}{2}\kappa x^2$. Find the location $x$ of the classical turning point in terms of $E$; use *twice* this distance for $L$; then insert this into the infinite well energy formula, so that $E$ appears on both sides. Thus far, the procedure really only deals with kinetic energy. Assume, as is true for a classical oscillator, that there is as much potential energy, on average, as kinetic energy. What do you obtain for the quantized energies?

**54.** The potential energy shared by two atoms in a diatomic molecule, depicted in Figure 17, is often approximated by the fairly simple function $U(x) = (a/x^{12}) - (b/x^6)$, where constants $a$ and $b$ depend on the atoms involved. In Section 7, it is said that near its minimum value, it can be approximated by an even simpler function—it should "look like" a parabola.

(a) In terms of $a$ and $b$, find the minimum potential energy $U(x_0)$ and the separation $x_0$ at which it occurs.
(b) The parabolic approximation $U_p(x) = U(x_0) + \frac{1}{2}\kappa(x - x_0)^2$ has the same minimum value at $x_0$ and the same first derivative there (i.e., 0). Its second derivative is $\kappa$, the spring constant of this Hooke's law potential energy. In terms of $a$ and $b$, what is the spring constant of $U(x)$?

## Section 8

**55.** Classically, if a particle is not observed, the probability per unit length of finding it in a box is a constant $1/L$ along the entire length of the box. With this, show that the classical expectation value of the position is $\frac{1}{2}L$, that the expectation value of the square of the position is $\frac{1}{3}L^2$, and that the uncertainty in position is $L/\sqrt{12}$.

**56.** Show that the uncertainty in a particle's position in an infinite well in the general case of arbitrary $n$ is given by

$$L\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}$$

Discuss the dependence. In what circumstance does it agree with the classical uncertainty of $L/\sqrt{12}$ discussed in Exercise 55?

**57.** Show that the uncertainty in a particle's momentum in an infinite well in the general case of arbitrary $n$ is given by $n\pi\hbar/L$.

**58.** What is the product of the uncertainties determined in Exercises 56 and 57? Discuss the result.

**59.** Determine the expectation value of the position of a harmonic oscillator in its ground state.

**60.** Show that the uncertainty in the position of a ground-state harmonic oscillator is $(1/\sqrt{2})(\hbar^2/m\kappa)^{1/4}$.

**61.** Show that the uncertainty in the momentum of a ground-state harmonic oscillator is $(\sqrt{\hbar/2})(m\kappa)^{1/4}$.

**62.** What is the product of the uncertainties determined in Exercises 60 and 61? Explain.

**63.** Repeat Exercises 60–62 for the first excited ($n = 1$) state of a harmonic oscillator.

**64.** If a particle in a stationary state is *bound*, the expectation value of its momentum must be 0. (a) In words, why? (b) Prove it. Starting from the general expression (31) with $\hat{p}$ in place of $\hat{Q}$, integrate by parts, then argue that the result is identically 0. Be careful that your argument is somehow based on the particle being *bound*; a free particle certainly may have a nonzero momentum. (*Note:* Without loss of generality, $\psi(x)$ may be chosen to be real.)

### Section 9

**65.** In equation (33), the two solutions are added in equal amounts. Show that if we instead added different percentages of the two solutions, it would not change the important conclusion related to the oscillation frequency of the charge density.

* **66.** Consider a wave function that is a combination of two different infinite well stationary states, the $n$th and the $m$th.

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\Psi_n(x)e^{-i(E_n/\hbar)t} + \frac{1}{\sqrt{2}}\Psi_m(x)e^{-i(E_m/\hbar)t}$$

(a) Show that $\Psi(x, t)$ is properly normalized.

(b) Show that the expectation value of the energy is the average of the two energies: $\overline{E} = \frac{1}{2}(E_n + E_m)$. (*Be careful:* The temporal part of the wave function definitely does *not* drop out.)

(c) Show that the expectation value of the square of the energy is given by

$$\overline{E^2} = \tfrac{1}{2}(E_n^2 + E_m^2)$$

(d) Determine the uncertainty in the energy.

### Section 10

See the Computational Exercises section.

### Section 11

**67.** Prove that the transitional-state wave function (33) does not have a well-defined energy.

**68.** To describe a matter wave, does the function $A\sin(kx)\cos(\omega t)$ have a well-defined energy? Explain.

**69.** Does the wave function $\psi(x) = A(e^{+ikx} + e^{-ikx})$ have a well-defined momentum? Explain.

**70.** The operator for angular momentum about the $z$-axis in spherical polar coordinates is $-i\hbar(\partial/\partial\phi)$. Find a function $f(\phi)$ that would have a well-defined $z$-component of angular momentum.

**71.** Show that $\Delta p = 0 \Rightarrow \hat{p}\psi(x) = \overline{p}\psi(x)$. That is, verify that unless the wave function is an eigenfunction of the momentum operator, there will be a nonzero uncertainty in momentum. Start by showing that the quantity

$$\int_{\text{all space}} \psi^*(x)(\hat{p} - \overline{p})^2\psi(x)dx$$

is $(\Delta p)^2$. Then, using the differential operator form of $\hat{p}$ and integration by parts, show that it is also

$$\int_{\text{all space}} [(\hat{p} - \overline{p})\psi(x)]^*[(\hat{p} - \overline{p})\psi(x)]dx$$

(*Note:* Because momentum is real, $\overline{p}$ is real.) Together these show that if $\Delta p$ is 0, then the preceding quantity must be 0. However, the integral of the complex square of a function (the quantity in brackets) can only be 0 if the function is identically 0, so the assertion is proved.

## Comprehensive Exercises

**72.** In a study of heat transfer, we find that for a solid rod, there is a relationship between the second derivative of the temperature with respect to position along the rod and the first with respect to time. (A *linear* temperature change with position would imply as much heat flowing into a region as out, so the temperature there would not change with time.)

$$\frac{\partial^2 T(x,t)}{\partial x^2} = b\frac{\partial T(x,t)}{\partial t}$$

(a) Separate variables. That is, assume a solution that is a product of a function of $x$ and a function of $t$,