GLOBAL EDITION

Fundamentals of Communication Systems

SECOND EDITION

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FUNDAMENTALS
OF COMMUNICATION SYSTEMS

The noise power is

$$\begin{aligned} P_n &= \int_{-\infty}^{\infty} S_n(f)\,df \\ &= \frac{N_0}{2} \times 4W \\ &= 2WN_0. \end{aligned} \tag{6.1.11}$$

Now we can find the output SNR as

$$\begin{aligned} \left(\frac{S}{N}\right)_o &= \frac{P_o}{P_{n_o}} \\ &= \frac{\frac{A_c^2}{4}P_M}{\frac{1}{4}2WN_0} \\ &= \frac{A_c^2 P_M}{2WN_0}. \end{aligned} \tag{6.1.12}$$

In this case, the received signal power, as given by Equation (3.2.2), is $P_R = \frac{A_c^2 P_M}{2}$. Therefore, the output SNR in Equation (6.1.12) for DSB-SC AM may be expressed as

$$\left(\frac{S}{N}\right)_{o_{\text{DSB}}} = \frac{P_R}{N_0 W}, \tag{6.1.13}$$

which is identical to $(S/N)_b$, which is given by Equation (6.1.2). Therefore, in DSB-SC AM, the output SNR is the same as the SNR for a baseband system. In other words, DSB-SC AM does not provide any SNR improvement over a simple baseband communication system.

6.1.3 Effect of Noise on SSB AM

In this case, the modulated signal, as given in Equation (3.2.8), is

$$u(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t). \tag{6.1.14}$$

Therefore, the input to the demodulator is

$$\begin{aligned} r(t) &= A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t) + n(t) \\ &= (A_c m(t) + n_c(t))\cos(2\pi f_c t) + (\mp A_c \hat{m}(t) - n_s(t))\sin(2\pi f_c t). \end{aligned} \tag{6.1.15}$$

Here we assume that demodulation occurs with an ideal phase reference. Hence, the output of the lowpass filter is the in-phase component (with a coefficient of $\frac{1}{2}$) of the preceding signal. That is,

$$y(t) = \frac{A_c}{2}m(t) + \frac{1}{2}n_c(t). \tag{6.1.16}$$

We observe that, in this case again, the signal and the noise components are additive, and a meaningful SNR at the receiver output can be defined. Parallel to our discussion of DSB, we have

$$P_o = \frac{A_c^2}{4} P_M \tag{6.1.17}$$

and

$$P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n, \tag{6.1.18}$$

where

$$P_n = \int_{-\infty}^{\infty} S_n(f)\, df = \frac{N_0}{2} \times 2W = W N_0. \tag{6.1.19}$$

Therefore,

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{A_c^2 P_M}{W N_0}. \tag{6.1.20}$$

But in this case,

$$P_R = P_U = A_c^2 P_M; \tag{6.1.21}$$

thus,

$$\left(\frac{S}{N}\right)_{o\text{SSB}} = \frac{P_R}{W N_0} = \left(\frac{S}{N}\right)_{\text{b}}. \tag{6.1.22}$$

Therefore, the signal-to-noise ratio in a single-sideband system is equivalent to that of a DSB system.

6.1.4 Effect of Noise on Conventional AM

In conventional DSB AM, the modulated signal was given in Equation (3.2.6) as

$$u(t) = A_c\,[1 + a m(t)] \cos 2\pi f_c t. \tag{6.1.23}$$

Therefore, the received signal at the input to the demodulator is

$$r(t) = [A_c\,[1 + a m_n(t)] + n_c(t)] \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t, \tag{6.1.24}$$

where a is the modulation index and $m_n(t)$ is normalized so that its minimum value is -1. If a synchronous demodulator is employed, the situation is basically similar to the DSB case, except that we have $1 + a m_n(t)$ instead of $m(t)$. Therefore, after mixing and lowpass filtering, we have

$$y_1(t) = \frac{1}{2}\,[A_c\,[1 + a m_n(t)] + n_c(t)]\,. \tag{6.1.25}$$

However, in this case, the desired signal is $m(t)$, not $1 + a m_n(t)$. The DC component in the demodulated waveform is removed by a DC block and, hence, the lowpass filter output is

$$y(t) = \frac{1}{2} A_c a m_n(t) + \frac{n_c(t)}{2}. \tag{6.1.26}$$

In this case, the received signal power is given by

$$P_R = \frac{A_c^2}{2}\left[1 + a^2 P_{M_n}\right], \tag{6.1.27}$$

where we have assumed that the message process is zero mean. Now we can derive the output SNR as

$$\begin{aligned}
\left(\frac{S}{N}\right)_{o_{\mathrm{AM}}} &= \frac{\frac{1}{4}A_c^2 a^2 P_{M_n}}{\frac{1}{4}P_{n_c}} \\
&= \frac{A_c^2 a^2 P_{M_n}}{2N_0 W} \\
&= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{\frac{A_c^2}{2}\left[1 + a^2 P_{M_n}\right]}{N_0 W} \\
&= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{P_R}{N_0 W} \\
&= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \left(\frac{S}{N}\right)_b \\
&= \eta \left(\frac{S}{N}\right)_b,
\end{aligned} \tag{6.1.28}$$

where we have used Equation (6.1.2) and η denotes the modulation efficiency.

We can see that, since $a^2 P_{M_n} < 1 + a^2 P_{M_n}$, the SNR in conventional AM is always smaller than the SNR in a baseband system. In practical applications, the modulation index a is in the range of 0.8–0.9. The power content of the normalized message process depends on the message source. For speech signals that usually have a large dynamic range, P_M is in the neighborhood of 0.1. This means that the overall loss in SNR, when compared to a baseband system, is a factor of 0.075 or equivalent to a loss of 11 dB. The reason for this loss is that a large part of the transmitter power is used to send the carrier component of the modulated signal and not the desired signal.

To analyze the envelope-detector performance in the presence of noise, we must use certain approximations. This is a result of the nonlinear structure of an envelope detector, which makes an exact analysis difficult. In this case, the demodulator detects the envelope of the received signal and the noise process. The input to the envelope detector is

$$r(t) = [A_c[1 + am_n(t)] + n_c(t)]\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t; \tag{6.1.29}$$

therefore, the envelope of $r(t)$ is given by

$$V_r(t) = \sqrt{[A_c[1 + am_n(t)] + n_c(t)]^2 + n_s^2(t)}. \tag{6.1.30}$$

Now we assume that the signal component in $r(t)$ is much stronger than the noise component. With this assumption, we have

$$P\left(n_c(t) \ll A_c\left[1 + am_n(t)\right]\right) \approx 1; \tag{6.1.31}$$

therefore, we have a high probability that

$$V_r(t) \approx A_c\left[1 + am_n(t)\right] + n_c(t). \tag{6.1.32}$$

After removing the DC component, we obtain

$$y(t) = A_c am_n(t) + n_c(t), \tag{6.1.33}$$

which is basically the same as $y(t)$ for the synchronous demodulation without the $\frac{1}{2}$ coefficient. This coefficient, of course, has no effect on the final SNR; therefore we conclude that, under the assumption of high SNR at the receiver input, the performance of synchronous and envelope demodulators is the same. However, if the preceding assumption is not true, we still have an additive signal and noise at the receiver output with synchronous demodulation, but the signal and noise become intermingled with envelope demodulation. To see this, let us assume that at the receiver input, the noise power[1] is much stronger than the signal power. This means that

$$\begin{aligned}
V_r(t) &= \sqrt{[A_c[1 + am_n(t)] + n_c(t)]^2 + n_s^2(t)} \\
&= \sqrt{A_c^2(1 + am_n(t))^2 + n_c^2(t) + n_s^2(t) + 2A_c n_c(t)(1 + am_n(t))} \\
&\overset{a}{\approx} \sqrt{\left(n_c^2(t) + n_s^2(t)\right)\left[1 + \frac{2A_c n_c(t)}{n_c^2(t) + n_s^2(t)}(1 + am_n(t))\right]} \\
&\overset{b}{\approx} V_n(t)\left[1 + \frac{A_c n_c(t)}{V_n^2(t)}(1 + am_n(t))\right] \\
&= V_n(t) + \frac{A_c n_c(t)}{V_n(t)}(1 + am_n(t)),
\end{aligned} \tag{6.1.34}$$

where (a) uses the fact that $A_c^2(1 + am_n(t))^2$ is small compared with the other components and (b) denotes $\sqrt{n_c^2(t) + n_s^2(t)}$ by $V_n(t)$, the envelope of the noise process; we have also used the approximation $\sqrt{1+\epsilon} \approx 1 + \frac{\epsilon}{2}$, for small ϵ, where

$$\epsilon = \frac{2A_c n_c(t)}{n_c^2(t) + n_s^2(t)}(1 + am_n(t)). \tag{6.1.35}$$

We observe that, at the demodulator output, the signal and the noise components are no longer additive. In fact, *the signal component is multiplied by noise* and is no longer distinguishable. In this case, no meaningful SNR can be defined. We say that this system *is operating below the threshold*. The subject of threshold and its effect on the performance

[1]By noise power at the receiver input, we mean the power of the noise within the bandwidth of the modulated signal or, equivalently, the noise power at the output of the noise-limiting filter.

of a communication system will be covered in more detail when we discuss the noise performance in angle modulation.

Example 6.1.2

We assume that the message is a wide-sense stationary random process $M(t)$ with the autocorrelation function

$$R_M(\tau) = 16\,\mathrm{sinc}^2(10{,}000\tau).$$

We also know that all the realizations of the message process satisfy the condition $\max|m(t)| = 6$. We want to transmit this message to a destination via a channel with a 50-dB attenuation and additive white noise with the power spectral density $\mathcal{S}_n(f) = \frac{N_0}{2} = 10^{-12}$ W/Hz. We also want to achieve an SNR at the modulator output of at least 50 dB. What is the required transmitter power and channel bandwidth if we employ the following modulation schemes?

1. DSB AM.
2. SSB AM.
3. Conventional AM with a modulation index equal to 0.8.

Solution First, we determine the bandwidth of the message process. To do this, we obtain the power spectral density of the message process, namely,

$$\mathcal{S}_M(f) = \mathscr{F}[R_M(\tau)] = \frac{16}{10{,}000}\Lambda\left(\frac{f}{10{,}000}\right),$$

which is nonzero for $-10{,}000 < f < 10{,}000$; therefore, $W = 10{,}000$ Hz. Now we can determine $\left(\frac{S}{N}\right)_b$ as a basis of comparison:

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{P_R}{2\times 10^{-12}\times 10^4} = \frac{10^8 P_R}{2}.$$

Since the channel attenuation is 50 dB, it follows that

$$10\log\frac{P_T}{P_R} = 50;$$

therefore,

$$P_R = 10^{-5} P_T.$$

Hence,

$$\left(\frac{S}{N}\right)_b = \frac{10^{-5}\times 10^8 P_T}{2} = \frac{10^3 P_T}{2}.$$

1. For DSB-SC AM, we have

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{10^3 P_T}{2} \sim 50 \text{ dB} = 10^5.$$

Therefore,

$$\frac{10^3 P_T}{2} = 10^5 \Longrightarrow P_T = 200 \text{ Watts}$$

and

$$\mathrm{BW} = 2W = 2\times 10{,}000 = 20{,}000 \quad \text{Hz} \approx 20 \text{ kHz}.$$

2. For SSB AM,

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{10^3 P_T}{2} = 10^5 \Longrightarrow P_T = 200 \text{ Watts}$$

and

$$\text{BW} = W = 10{,}000 \quad \text{Hz} = 10 \text{ kHz}.$$

3. For conventional AM, with $a = 0.8$,

$$\left(\frac{S}{N}\right)_o = \eta \left(\frac{S}{N}\right)_b = \eta \frac{10^3 P_T}{2},$$

where η is the modulation efficiency given by

$$\eta = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}}.$$

First, we find P_{M_n}, the power content of the normalized message signal. Since $\max |m(t)| = 6$, we have

$$P_{M_n} = \frac{P_M}{(\max |m(t)|)^2} = \frac{P_M}{36}.$$

To determine P_M, we have

$$P_M = R_M(\tau)_{|\tau=0} = 16;$$

therefore,

$$P_{M_n} = \frac{16}{36} = \frac{4}{9}.$$

Hence,

$$\eta = \frac{0.8^2 \times \frac{4}{9}}{1 + 0.8^2 \times \frac{4}{9}} \approx 0.22.$$

Therefore,

$$\left(\frac{S}{N}\right)_o \approx 0.22 \frac{10^3 P_T}{2} = 0.11 \times 10^3 P_T = 10^5$$

or

$$P_T \approx 909 \text{ Watts}.$$

The bandwidth of conventional AM is equal to the bandwidth of DSB AM, i.e.,

$$\text{BW} = 2W = 20 \text{ kHz}.$$

■

6.2 EFFECT OF NOISE ON ANGLE MODULATION

In this section, we will study the performance of angle-modulated signals when contaminated by additive white Gaussian noise; we will also compare this performance with the performance of amplitude-modulated signals. Recall that in amplitude modulation, the