

INTERNATIONAL EDITION

Introduction to Finite Elements in Engineering

Fourth Edition

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ALWAYS LEARNING

PEARSON

Introduction to Finite Elements in Engineering

FOURTH EDITION

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PROBLEMS

4.1. Consider the truss element shown in Fig. P4.1. The x-, y-coordinates of the two nodes are indicated in the figure. If $\mathbf{q} = [2, 0.7, 2.5, 5] \times 10^{-2}$ in., determine the following:

(a) the vector $\mathbf{q}'$
(b) the stress in the element
(c) the $\mathbf{k}$ matrix
(d) the strain energy in the element.

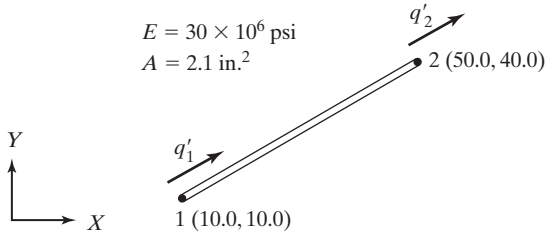


FIGURE P4.1

4.2. A truss element, with local node numbers 1 and 2, is shown in Fig. P4.2.

(a) What are the direction cosines ℓ and m.
(b) Show the x'-axis, $q_1, q_2, q_3, q_4, q'_1, q'_2$ on the figure.
(c) If $\mathbf{q} = [0., 0.01, -0.025, -0.05]^{\mathrm{T}}$, determine q'_1, q'_2.

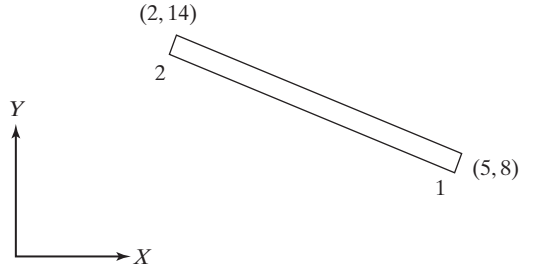


FIGURE P4.2

4.3. For the pin-jointed configuration shown in Fig. P4.3, determine the stiffness values K_{11}, K_{12}, and K_{22} of the global stiffness matrix.

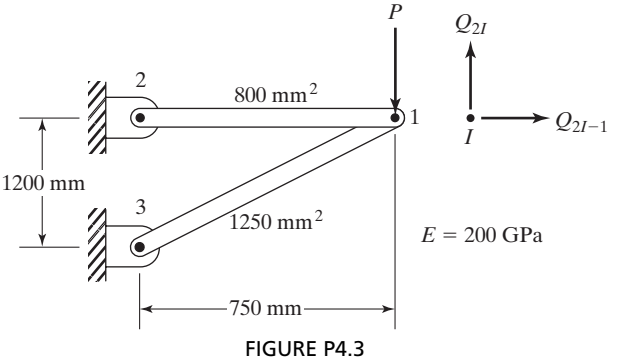


FIGURE P4.3

4.4. For the truss in Fig. P4.4, a horizontal load of $P = 2500$ lb is applied in the x direction at node 2.

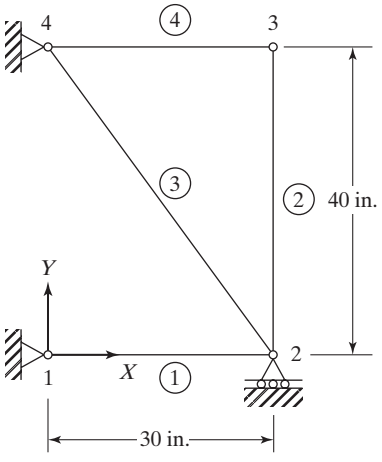


FIGURE P4.4

(a) Write down the element stiffness matrix **k** for each element.
(b) Assemble the **K** matrix.
(c) Using the elimination approach, solve for **Q**.
(d) Evaluate the stress in elements 2 and 3.
(e) Determine the reaction force at node 2 in the y direction.

4.5. For Fig. P4.4, determine the stresses in each element due to the following support movement: support at node 2, 0.36 in. down.

4.6. For the two-bar truss shown in Fig. P4.6, determine the displacements of node 1 and the stress in elements 1–3.

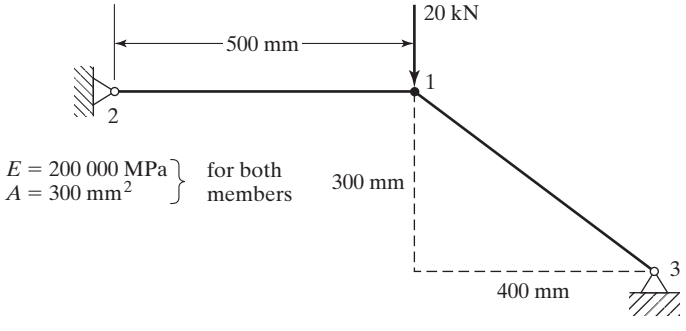


FIGURE P4.6

4.7. For the three-bar truss shown in Fig. P4.7, determine the displacements of node 1 and the stress in element 3.

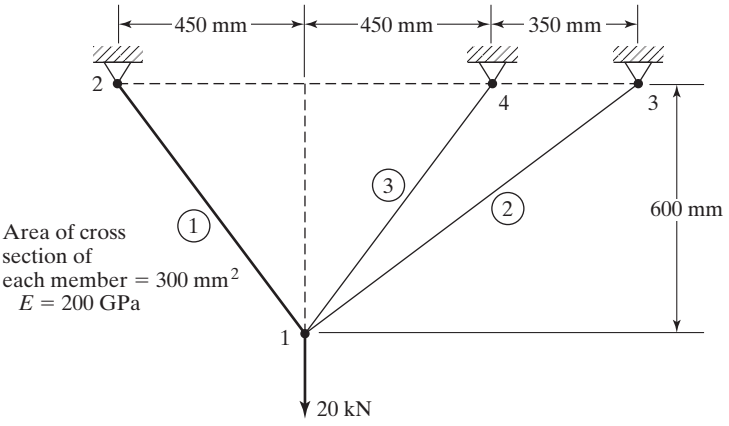


FIGURE P4.7

4.8. For the 2-D truss configuration shown in Fig. P4.8, determine the bandwidth for stiffness storage in a banded form. Choose an alternative numbering scheme and determine the corresponding bandwidth. Comment on the strategy that you use for decreasing the bandwidth.

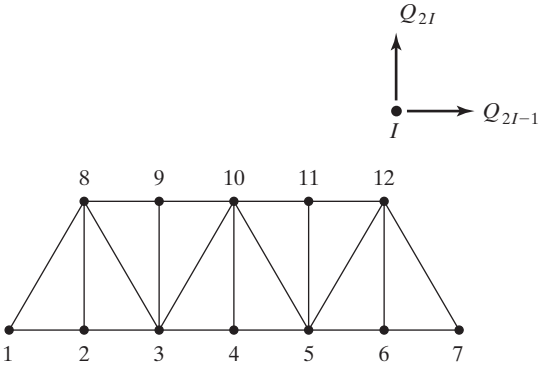


FIGURE P4.8

4.9. A small railroad bridge is constructed of steel members, all of which have a cross-sectional area of 3000 mm^2. A train stops on the bridge, and the loads applied to the truss on one side of the bridge are as shown in Fig. P4.9. Estimate how much the point R moves horizontally because of this loading. Also determine the nodal displacements and element stresses.

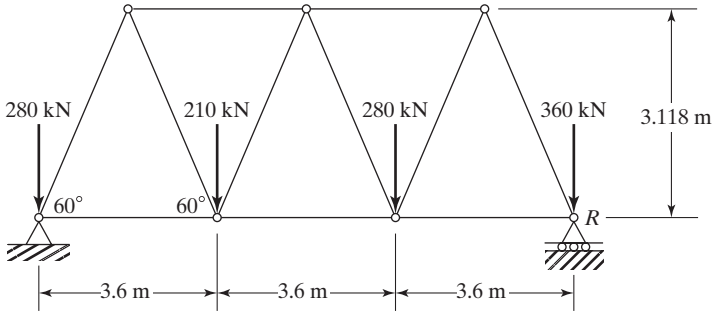


FIGURE P4.9

4.10. Consider the truss in Fig. P4.10 loaded as shown. Cross-sectional areas in square inches are shown in parentheses. Consider symmetry and model only one-half of the truss shown. Determine displacements and element stresses. Let $E = 30 \times 10^6$ psi.

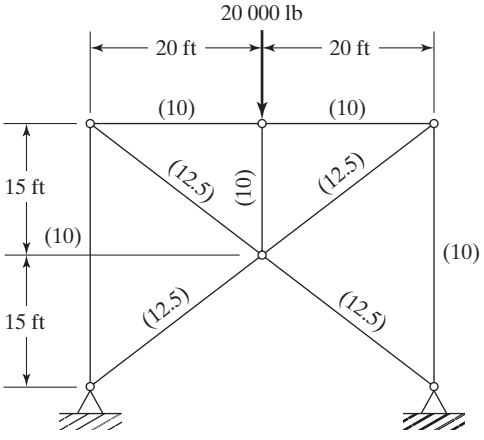


FIGURE P4.10

4.11. Determine the nodal displacements and element stresses in the truss in Fig. P4.11, due to each of the following conditions:

(a) Increase of temperature of 50°F in elements 1, 3, 7, and 8.

(b) Elements 9 and 10 are $\frac{1}{4}$ in. too short and element 6 is $\frac{1}{8}$ in. too long, owing to errors in fabrication, and it was necessary to force them into place.

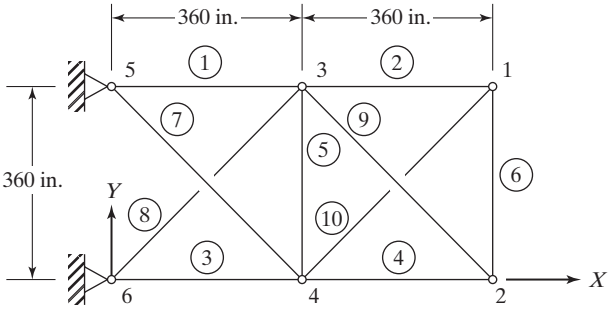


FIGURE P4.11

(c) Support at node 6 moves 0.12 in. down. *Data*: Take $E = 30 \times 10^6$ psi, $\alpha = 1/150\ 000$ per °F. Cross-sectional areas for each element are as follows:

Element	Area (in.^{2})
1, 3	25
2, 4	12
5	1
6	4
7, 8, 9	17
10	5

4.12. A two-member truss is subjected to a load $P = 8000$ N. Member 1–2 is 400 mm long. Member 1–3 was manufactured to be 505 mm long instead of 500 mm. However, it was forced into place. Determine

(a) the stresses in the members assuming that member 1–3 was manufactured to its correct length of 500 mm

(b) the stresses in the members as a result of member 1–3 being forced into place (and the load P, of course).

(*Hint* : Treat this as an initial strain problem and use the temperature load vector expression in the text.) Take cross-sectional areas = 750 mm^2, $E = 200$ GPa.

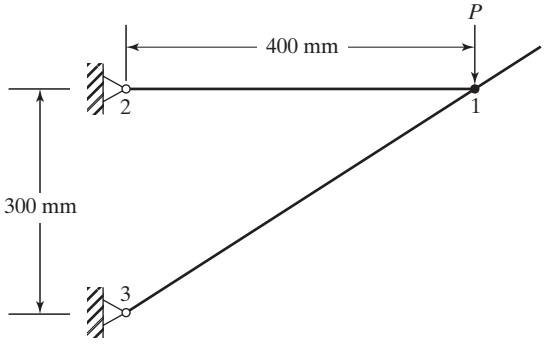


FIGURE P4.12

4.13. Expressions for the element stress (Eq. 4.16), and element temperature load (Eq. 4.22) were derived for a 2-D truss element. Generalize these expressions for a 3-D truss element.

4.14. Find deflections at nodes, stresses in members, and reactions at supports for the truss shown in Fig. P4.14 when the 200-kip load is applied.

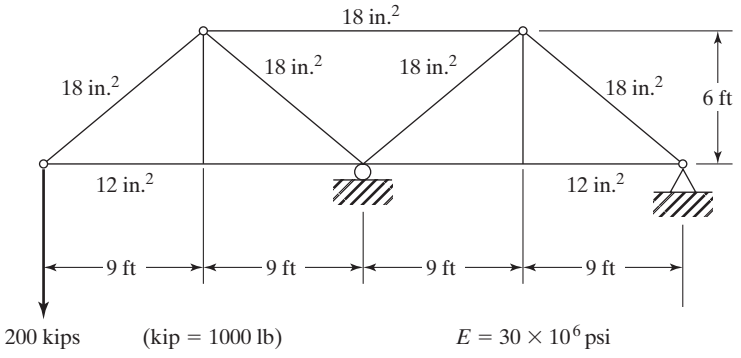


FIGURE P4.14

4.15. Find the deflections at the nodes for the truss configuration shown in Fig. P4.15. Area = 8 in.^{2} for each member.

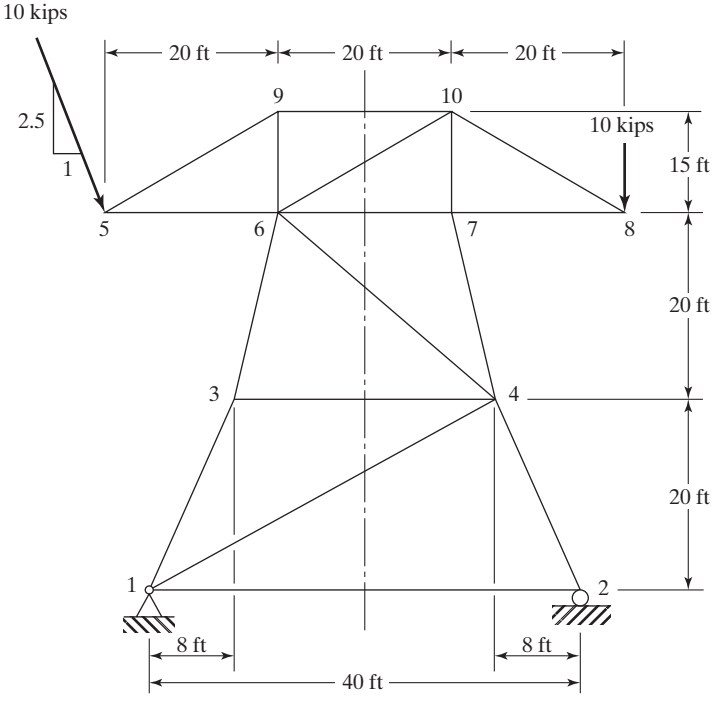


FIGURE P4.15