



Traditional IQ tests may disadvantage children who take novel or creative approaches to solving problems. Creativity and divergent thinking are often associated with highly intelligent or gifted children, yet this type of thinking may lead to lower scores on traditional tests of IQ.

Alternative perspectives suggest that tests of verbal and numerical ability should be considered to be measuring fundamentally different types of intelligence. However, critics ask where we should draw the line in associating particular abilities with different intelligences. More recently, theories have been developed which suggest that emotional intelligence should be considered another kind of intelligence. Supporters of multiple intelligences point out that this way of thinking allows us to acknowledge individual differences which exist between children in terms of their strengths and weaknesses, instead of labelling children as above or below some normal level of intelligence.

Despite developmental psychology's focus on childhood, we have seen that in both of these areas of cognition there are changes through adulthood as well. Research suggests that we reach the peak of our memory abilities in young adulthood, at which point it appears to plateau before declining in older adulthood. For adults who remain physically healthy, a general deterioration in memory may be due to loss of ability in any of the three sub-systems. However, recent research seems to point to encoding as the part of memory where a decline in ability is particularly apparent, because older adults appear less able to benefit from the use of memory strategies.

Mental age increases through childhood and then plateaus in the mid-teenage years, whilst chronological age continues to increase right through the lifespan. When calculating IQ, adults will therefore be at a disadvantage as they get older. This has led to the development of intelligence tests specifically for adults. Twin studies have led researchers to believe that, whilst our genetic inheritance is largely responsible for intelligence, the influence of the environment is still important. So intelligence comes about as the result of interaction between these two factors.

REVIEW QUESTIONS

- 1. Compare and contrast three different types of memory which children may have.
- 2. Explain the encoding specificity principle, and describe the changes in encoding specificity in children's development.
- 3. Describe one method designed to research infant memory.
- 4. In what ways might top-down processing affect children's memory?
- 5. What is a cognitive interview? Explain the rationale for its different components.
- 6. How many different intelligences do you think there are, and why?
- 7. What are the pros and cons of using traditional IQ tests to measure intelligence?

RECOMMENDED READING

For a comprehensive review of the research on infant memory development specifically, see:

Hayne, H. (2004). Infant memory development: Implications for childhood amnesia. Developmental Review, 24, 33-73.

This paper also contains a useful review of comparisons between adult memory research and infant memory research, which may help to answer these questions.

To read other research about the development of children's mathematical abilities and the role of working memory, see:

Barrouillet, P., & Lepine, R. (2005). Working memory and children's use of retrieval to solve addition problems. *Journal of Experimental Child Psychology*, 91, 183–204.

For a thorough examination of the development of children's ability to attend selectively, see:

Miller, P. H. (1990). The development of strategies of selective attention. In D. F. Bjorklund (Ed.), *Children's Strategies: Contemporary Views of Cognitive Development* (157–184). Hillsdale, NJ: Erlbaum.

For more information on the field of face recognition in children, see:

Pascalis, O., & Slater, A. (Eds.) (2003). The Development of Face Processing in Infancy and Early Childhood: Current Perspectives. New York: Nova Science.

For a consideration of memory research and how it helps us to understand the phenomenon of infantile amnesia, see:

Hayne, H. (2004). Infant memory development: Implications for childhood amnesia. *Developmental Review*, 24, 33–73.

For an overview of the foundations of research into the role of scripts in the development of memory, see:

Nelson, K., & Gruendel, J. (1986). Children's scripts. In K. Nelson (Ed.), *Event Knowledge, Structure and Function in Development* (231–247). Hillsdale, NJ: Erlbaum.

For further reading about false memories, see:

Brainerd, C. J., & Reyna, V. F. (2005). *The Science of False Memories*. Oxford: Oxford University Press.

For a detailed exploration of the development of cognitive interviewing as a technique for improving the witness testimony of children, see:

Fisher, R. P., & Geiselman, R. E. (1992). *Memory Enhancing Techniques for Investigative Interviewing: The Cognitive Interview*. Springfield, IL: Charles C. Thomas.

Geiselman, R. E., & Fisher, R. P. (1997). Ten years of cognitive interviewing. In D. G. Payne & F. G. Conrad (Eds.), *Intersections in Basic and Applied Memory Research* (291–310). Mahwah, NJ: Erlbaum.

For a consideration of the influences of heredity and the environment on cognition right across the lifespan, see:

McGue, M., Bouchard, T. J., Iacono, W. G., & Lykken, D. T. (1993). Behavioural genetics and cognitive ability: A lifespan perspective. In R. Plomin & G. E. McClearn (Eds.), *Nature, Nurture and Psychology* (59–76). Washington: APA.

RECOMMENDED WEBSITES

Human memory: test yourself:

http://www.psychologistworld.com/memory/test1.php

Cognitive interviewing: a 'how-to' guide:

http://appliedresearch.cancer.gov/archive/cognitive/interview.pdf#search=cognitive%20interview

Mnemonics: fun with words:

http://www.fun-with-words.com/mnemonics.html

The mystery of infant memories:

http://brainconnection.brainhq.com/2013/04/22/gone-but-not-forgotten-the-mystery-behind-infant-memories/

fMRI:

http://www.radiologyinfo.org/en/info.cfm?pg=fmribrain&bhcp=1

Take an IQ test online:

http://www.free-iqtest.net/iq.asp

National Association for Able Children in Education:

http://www.nace.co.uk/

Chapter 7

The development of mathematical thinking

Terezinha Nunes and Peter Bryant

Learning outcomes

After reading this chapter, and with further recommended reading, you should be able to:

- 1. Explain the difference between mathematical concepts and categorical concepts.
- 2. Discuss the difficulties in learning the meaning of numbers, both whole and rational numbers, and the connections between numbers and quantities.
- 3. Describe research into children's learning of number meanings, including whole and rational numbers.
- 4. Explain the connection between schemas of action and children's understanding of arithmetic operations.
- 5. Design theoretically driven assessments of children's understanding of additive and multiplicative reasoning.
- 6. Use research to argue that there is a crucial difference between knowing how and knowing when to do sums.





The power of maths

It is most likely that you have seen this picture in other psychology texts. It is called the Müller-Lyer illusion (Müller-Lyer, 1890). The horizontal line is divided in two segments, *a* and *b*, by the arrow head. Are the segments the same size? When you look at them, you see them as different, but if you take a ruler and measure them, you will find that they are the same. This is an example of the power and also of the intellectual demands of mathematics.

The power of mathematics in this example is that it allows you to go beyond perception: you don't see the segments as the same, but you know that they are the same. You use something to represent the size of the segments, compare the representations and draw a conclusion from the representations. Much of mathematics is about using signs, such as numbers or letters, to represent something like a quantity, a relation or an unknown value, and about manipulating these signs to arrive at conclusions about what is represented. Mathematics therefore allows us to go beyond mere perception in understanding the world around us.

- What numbers, letters or symbols can you think of that are used in mathematics?
- Think back across the course of the last couple of days. How many times have you used maths in your everyday life?
- What problems, questions or challenges has maths allowed you to solve?

Introduction

Mathematical thinking has been of interest to developmental psychologists for about a century and it still fascinates cognitive development researchers. There are many reasons for the continued interest in how children's mathematical thinking develops. Some people become interested in how children learn mathematics because they are, themselves, very fond of mathematics. Others are fascinated by children's mathematical thinking because of the ingenious ways in which some children solve mathematical problems, drawing on resources that might not have occurred to us adults. Still others are interested in children's mathematical thinking because of the amazing misconceptions and deductive failures to which children seem to be prone.

Besides all of these somewhat personal reasons, there is one really good reason for developmental psychologists to be interested in mathematics: mathematics is such an important part of our culture that everyone who goes to school is required to learn mathematics. The universal aim of learning mathematics is matched only by one other learning aim, which is learning to read and write. It is obvious why every child is required to learn to read and write: reading gives people access to books and to the internet, from which we can all learn a great deal, and it enables us to learn on our own. Reading gives us a ticket to ideas and information, ways of thinking and arguing, and communication that is not bound by things that are immediately present in our physical environment.

Most people would readily agree that mathematics is just as necessary for everyone as reading, but their reasons may differ. Some



If all we needed to know about maths were computations then we wouldn't need much more than just a calculator.

people think that mathematics is important for everyone because it is a skill that we all need in daily life. People need to know the arithmetic operations required to get on in life. For example, we need to calculate prices and change; to estimate how long we need to get somewhere; to determine when we should leave the house if we do not want to be late for a meeting; to figure out how much medicine to give our pet if the dosage depends on body weight.

Others might argue that, whilst all of this is true, it is not the whole story. Mathematical skills are certainly important for daily life, but mathematics is much more than this, and everyone needs much more than computation skills. Galileo (whose father did not want him to become a mathematician because this was such a poorly paid career) saw mathematics as a way of understanding

this great book of the universe, which stands continuously open to our gaze [but which] cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics. (Sobel, 2000, p. 16)

If all we needed to know about mathematics were computation skills, we would not need to learn mathematics nowadays: anyone can afford a calculator today, and this would solve the problem. However, each time we use a calculator, we need to make a choice about which calculation is the right one to use. The understanding behind this choice is much more than a skill; it involves insight into the relationship between numbers and the world. This chapter is about how children develop insights into the relationships between numbers and the world; it is not about calculation skills. It encompasses calculation only in so far as the way in which children calculate tells us something about the way they think about, and with, numbers.

CASE STUDY

The complexity of understanding relations

Megan, Alice and Deborah, aged 4, meet in the playground. Alice points to a boy on a tricycle and says: 'That's my brother.' Deborah points to a boy climbing a frame and says: 'That's my brother'. This is the first time Megan has come across the word 'brother'. Megan's mum comes over to give her a drink. Megan says, as she points to the correct boys: 'Mummy, that is Alice's brother and that is Deborah's brother'. She used the word correctly.

- Does Megan know what the word 'brother' means?
- What is difficult about the meaning of the word 'brother'?
- In what way is it different from words like 'cat' and 'car'?

What is mathematical thinking?

The development of mathematical thinking is in some ways similar (but of course not identical) to language learning. In order to progress in mathematical thinking, children must learn mathematical symbols and their meanings, and must be able to combine them sensibly, just as they learn to combine words sensibly in sentences. Learning meanings for mathematical symbols is often more difficult than one might expect. Think of learning the meaning of the word 'brother', in the example above. Megan was able to tell her mother correctly that the two boys in the playground were Alice's and Deborah's brothers. But this does not necessarily mean that she knows the meaning of 'brother'. 'Brother of' is an expression that is based on a set of kinship relationships; in order to understand its meaning, we need to understand other relationships, including 'mother of' and 'father of'. We could test whether Megan really understood the meaning of the word 'brother' in several different ways. For example, if she found out that another little girl in the playground, Emma, was Alice's sister, she should be able to know that Alice's brother is also Emma's brother.

Think back to the Müller-Lyer illusion from the beginning of the chapter, which illustrates how we can use mathematics to understand space. The intellectual demand here is that you need to understand **transitivity**. In other words, if two elements *A* and *B* have a relation

to each other, and B and C also have the same relation, this relation is transitive if it implies that A and C also have the same relation to each other. In the illusion, if a is equal to 3.3 cm and b is equal to 3.3 cm, then a = b. Equality and equivalence are transitive relations. (Note, however, that if $A \neq B$ and $B \neq C$, this does not imply that $A \neq C$. Non-equivalence is not a transitive relation.) This illustrates the core intellectual demand that mathematics makes of us: the need to understand relations between things, rather than just understanding things in isolation.

Understanding relational concepts, such as 'brother of', is different from learning the meanings of words like 'cat' and 'car' for two main reasons. Firstly, when children learn words like 'cat' and 'car', they develop perceptual **prototypes** for cats and cars and use these prototypes to identify other exemplars of these categories. Chapter 5, Language Development, explores the

Definitions

Transitivity: a property of relations where new logical conclusions about one relation can be reached on the basis of premises about two other relations. For example, if A = B and B = C, then A = C. **Relations:** the positions, associations, connections, or status of one person, thing or quantity with regard to another or others; relational statements have an implied converse, so that (for example) if A is greater than B, then B must be less than A. **Prototype:** a person or a thing that serves as an example of a type.