Water-Resources Engineering

THIRD EDITION

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PEARSON

WATER-RESOURCES ENGINEERING

Third Edition

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TABLE 7.5: Calculation of Headwater Depths for Various Flow Types

Type	Equation number	Notes
1 & 2	7.11	Δh has different meaning for Type 1 and Type 2 flows
3	7.14 or 7.15	Equation 7.15 is preferred
4	7.20	—
5	7.22 or 7.26	Form 1 of Equation 7.26 is preferred; Form 2 is second
6	7.29	—

7.2.3.2 Fixed-flow method

Some approaches to culvert design calculate the required headwater depth, H, to pass a given design flow rate, Q, through a culvert of diameter, D, for given tailwater conditions. In this approach, the headwater depth, H, required for each type of flow is calculated and then the available headwater depth must be sufficient to accommodate the maximum H under design flow conditions. The equations used to calculate H for each flow type are given in Table 7.5. This design approach of finding H for given Q and D is in widespread use, and the Type 3 and Type 5 culvert equations expressed in terms of H/D are particularly suited to this approach. It is also useful to recall that Type 3 and Type 5 flows are under inlet control, which means that inlet conditions alone control the flow rate, Q, through the culvert. Conversely, Types 1, 2, 4, and 6 flows are under outlet control, which means that both inlet and outlet conditions determine Q.

The procedure used in the fixed-flow method of calculating H for a given Q and D is similar to that used in the fixed headwater method and can be summarized as follows:

Step 1: Assume a possible flow type and calculate H. If the calculated H is consistent with the assumed flow type, then the calculated H is the actual headwater depth. If not, go to Step 2.

Step 2: Repeat Step 1 for each possible flow type until the calculated H is consistent with the assumed flow type. If none of the possible flow types are confirmed, then take H as the maximum of all calculated values of H.

EXAMPLE 7.3

A 915-mm-diameter concrete culvert is 20 m long and is laid on a horizontal slope. The culvert entrance is flush with the headwall with a grooved end and the estimated entrance loss coefficient is 0.2. The design flow rate is 1.70 m^3/s and under design conditions the tailwater depth is 0.75 m. Estimate the headwater depth required for the culvert to accommodate the design flow rate.

Solution From the given data: $D = 0.915$ m, $Q = 1.70$ m^3/s, $L = 20$ m, $S_0 = 0$, $k_e = 0.2$, and TW $= 0.75$ m. For a concrete culvert it can be assumed that $n = 0.013$. Since the culvert is horizontal, $y_n = \infty$ and since the exit is not submerged the only possible flow regimes are Types 2, 3, and 6. These are considered sequentially as follows:

Type 2 Flow: For Type 2 flow, the difference between the headwater elevation and the crown of the culvert exit, Δh, is given by Equation 7.11. From the given data,

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.915)^2 = 0.6576 \text{ m}^2$$

$$V = \frac{Q}{A} = \frac{1.70}{0.6576} = 2.585 \text{ m/s}$$

$$R = \frac{D}{4} = \frac{0.915}{4} = 0.2288 \text{ m}$$

and substituting into Equation 7.11 gives

$$\Delta h = \frac{n^2V^2L}{R^{\frac{4}{3}}} + k_e\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$H - D = \frac{(0.013)^2(2.585)^2(20)}{(0.2288)^{\frac{4}{3}}} + 0.2\frac{(2.585)^2}{2(9.81)} + \frac{(2.585)^2}{2(9.81)} = 0.570 \text{ m}$$

where H is the headwater depth. The calculated result that $H - D = 0.570$ m validates the assumption of Type 2 flow, and gives

$$H = D + 0.570 \text{ m} = 0.915 \text{ m} + 0.570 \text{ m} = 1.485 \text{ m}$$

It is noteworthy that the calculated value of $H - D$ will always be positive; therefore if the culvert is hydraulically long ($L > 10D$) a horizontal slope and an unsubmerged outlet will always support Type 2 flow. Type 2 is not the only possible type of flow, since Type 3 flow might be supported in cases where the culvert is hydraulically short ($L < 10D$), and Type 6 flow might also be possible in cases where the entrance is not submerged. If the tailwater elevation was very low (not in this case), Type 5 flow would also be a possibility. The other possible flow types are considered below.

Type 3 Flow: For Type 3 flow, the headwater depth can be calculated using Equation 7.15, which requires that Fr > 0.7. In this case

$$\text{Fr} = \frac{V}{\sqrt{gD}} = \frac{2.585}{\sqrt{(9.81)(0.915)}} = 0.863$$

Therefore, application of Equation 7.15 is validated. For a culvert entrance flush with the headwall and with a grooved end, Table 7.1 gives $c = 0.0292$ and $Y = 0.74$. Substituting into Equation 7.15 gives

$$\frac{H}{D} = 32.2\, c\, \text{Fr}^2 + Y - 0.5S_0$$

$$\frac{H}{0.915} = 32.2(0.0292)(0.863)^2 + 0.74 - 0.5(0)$$

which yields $H = 1.318$ m. This result indicates that Type 3 flow will require a headwater depth of 1.318 m. However, Type 3 flow is very unlikely because the culvert is hydraulically long ($L > 10D$) and horizontal ($y_n = \infty$), so the flow will most likely expand and fill the culvert before reaching the exit, thus attaining Type 2 flow.

Type 6 Flow: For Type 6 flow, the headwater depth is calculated using Equation 7.29. Neglecting the headwater velocity, Equation 7.29 can be expressed as

$$\Delta h + \frac{V_1^2}{2g} - \frac{V^2}{2g} = h_i + h_f$$

$$(H - \text{TW}) + 0 - \frac{Q^2}{2g\overline{A}^2} = k_e\frac{Q^2}{2g\overline{A}^2} + \left(\frac{nQ}{\overline{A}\,\overline{R}^{\frac{2}{3}}}\right)^2 L \qquad (7.32)$$

where $\overline{A}$ and $\overline{R}$ represent the average flow area and hydraulic radius at the entrance and exit of the culvert. Equation 7.32 is an implicit equation for the headwater depth, H, since $\overline{A}$ and $\overline{R}$ will also depend on H. Any attempt to solve this equation numerically will show that there is no solution for $H \leq D$ and so Type 6 flow is not possible.

Collectively, the results presented here have demonstrated that Type 2 flow is the likely regime, and this will require a headwater depth of 1.485 m when the culvert is passing the design flow rate.

7.2.3.3 Minimum-performance method

Both the fixed-headwater and the fixed-flow methods involve the two-step procedure of: (1) assume a flow type; and (2) validate the flow type. A commonly used approximate method that is widely advocated (e.g., ASCE, 2006; USFHWA, 2012) is to skip the validation step and simply use the most conservative value of the calculated flow rate or headwater as the design value. This approach has the potential to over design the culvert structure.

EXAMPLE 7.4

How would the required headwater depth in Example 7.3 change if the minimum-performance method were used?

Solution The possible flow regimes are Types 2, 3, and 6, and the calculated headwater depths for these cases are as follows:

Type	Headwater depth (m)
2	1.485
3	1.318
6	N/A

Based on these results, the minimum-performance method would require a headwater depth of 1.485 m corresponding to Type 2 flow. In this case the minimum-performance method yields the same result as the exact method where the validity of the assumed flow type was also determined.

7.2.4 Roadway Overtopping

In cases where the culvert headwater elevation exceeds the roadway crest elevation (i.e., the roadway is overtopped), the flow must be partitioned between flow through the culvert and flow over the roadway. Under overtopping conditions, the roadway is typically assumed to perform like a rectangular weir, in which case the flow rate over the roadway, Q_r, is given by

$$\boxed{Q_r = C_d L H_r^{\frac{3}{2}}} \tag{7.33}$$

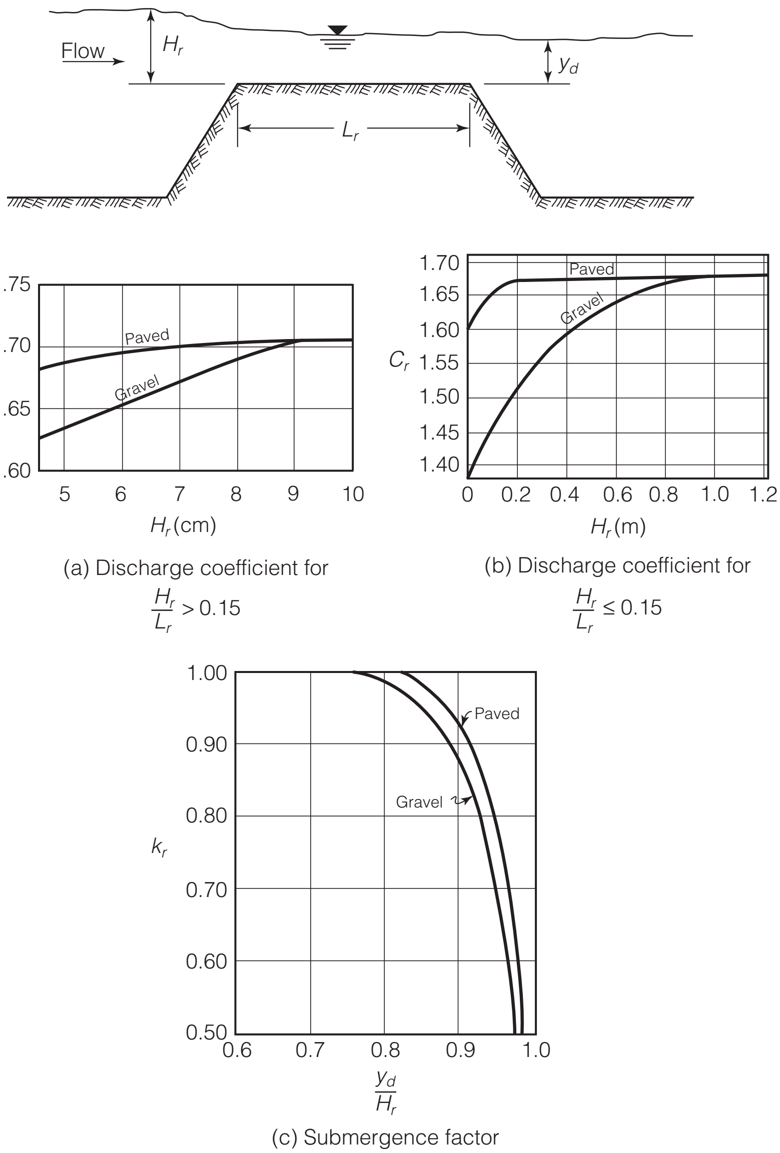
where C_d is the discharge coefficient, L is the roadway length over the culvert, and H_r is the head of water over the crest of the roadway. The discharge coefficient, C_d, can be estimated from the head of water over the roadway (H_r), the width of the roadway (L_r), and the submergence depth downstream of the roadway (y_d), using the relations shown in Figure 7.6, where the discharge coefficient is expressed in the form

$$C_d = k_r C_r \tag{7.34}$$

where C_r is derived from H_r using either Figure 7.6(a) for $H_r/L_r > 0.15$ or Figure 7.6(b) for $H_r/L_r \leq 0.15$, and k_r is derived from Figure 7.6(c) for a given value of y_d/H_r. Values of C_r and k_r derived from Figure 7.6 are used to calculate C_d using Equation 7.34, and this value of C_d is used in Equation 7.33 to calculate the flow rate over the roadway. Application of the weir equation (Equation 7.33) to describe the flow over a roadway does not take into account the effect of rails on the sides of the roadway. In cases of overflowing bridges, rails have been found to have a significant effect of the head-discharge relationship under overflow conditions (Klenzendorf and Charbeneau, 2009).

An iterative approach is usually required to determine the division of flow between the culvert and the roadway. This requires that different headwater elevations be assumed until the sum of the flow rates through the culvert and over the roadway is equal to the given total flow rate to be accommodated by the system.

FIGURE 7.6: Discharge coefficient for roadway overtopping
Source: USFHWA (2012).



EXAMPLE 7.5

A culvert under a roadway is to be designed to accommodate a 100-year peak flow rate of 2.49 m^3/s. The invert elevation at the culvert inlet is 289.56 m, the invert elevation at the outlet is 288.65 m, and the length of the culvert is to be 22.9 m. The channel downstream of the culvert has a rectangular cross section with a bottom width of 1.5 m, a slope of 4%, and a Manning's n of 0.045. The paved roadway crossing the culvert has a length of 15.2 m, an elevation of 291.08 m, and a width of 18.3 m. Considering a circular reinforced concrete pipe (RCP) culvert with a diameter of 610 mm and a conventional square-edge inlet and headwall, determine the depth of water flowing over the roadway, the flow rate over the roadway, and the flow rate through the culvert.

Solution For the given design flow rate, the tailwater elevation can be derived from the normal-flow condition in the downstream channel. Characteristics of the rectangular downstream channel are given as: $b = 1.5$ m, $S_0 = 0.04$, and $n = 0.045$. Taking $Q = 2.49$ m^3/s, the Manning equation gives

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

$$2.49 = \frac{1}{0.045}(1.5y_n)\left(\frac{1.5y_n}{1.5 + 2y_n}\right)^{\frac{2}{3}}(0.04)^{\frac{1}{2}}$$

which yields a normal flow depth, $y_n = 0.73$ m. Since the invert elevation of the downstream channel at the culvert outlet is 288.65 m, the tailwater elevation, TW, under the design condition is given by

$$\text{TW} = 288.65 \text{ m} + 0.73 \text{ m} = 289.38 \text{ m}$$

Since the diameter of the culvert is 0.61 m and the tailwater depth is 0.73 m, the culvert outlet is submerged; and since the roadway elevation is 291.08 m and the tailwater elevation is 289.38 m, the tailwater is below the roadway. Assuming that roadway overtopping (by the headwater) occurs under the design condition, the design flow rate is equal to the sum of the flow rate through the culvert and the flow rate over the roadway such that

$$Q = A\sqrt{\frac{2g\Delta h}{\frac{2gn^2L}{R^{\frac{4}{3}}} + k_e + 1}} + C_d L_R H_r^{\frac{3}{2}} \tag{7.35}$$

where Type 1 flow through the culvert exists (see Equation 7.12). From the given data: $Q = 2.49$ m^3/s, $D = 0.61$ m, $A = \pi D^2/4 = 0.292$ m^2, $n = 0.012$ (Table 7.3 for concrete pipe, good joints, smooth walls), $L = 22.9$ m, $R = D/4 = 0.153$ m, $k_e = 0.5$ (Table 7.4 for headwall, square edge), $L_R = 15.2$ m, and

$$\begin{aligned}\Delta h &= (\text{Roadway elevation} + H_r) - \text{Tailwater elevation}\\ &= (291.08 + H_r) - 289.38\\ &= 1.70 + H_r\end{aligned} \tag{7.36}$$

Combining Equations 7.35 and 7.36 with the given data yields

$$2.49 = 0.292\sqrt{\frac{2(9.81)(1.70 + H_r)}{\frac{2(9.81)(0.012)^2(22.9)}{(0.153)^{\frac{4}{3}}} + 0.5 + 1}} + C_d(15.2)H_r^{\frac{3}{2}}$$

which simplifies to

$$2.49 = 0.855\sqrt{1.70 + H_r} + 15.2C_d H_r^{\frac{3}{2}} \tag{7.37}$$

The discharge coefficient, C_d, depends on the head over the roadway, H_r, via the graphical relations in Figure 7.6. Taking $L_r = 18.3$ m and $y_d = 0$ (since the tailwater is below the roadway), the simultaneous solution of Equation 7.37 and the graphical relations in Figure 7.6 is done by iteration in the following table:

(1) H_r (m)	(2) H_r/L_r	(3) C_r	(4) y_d/H_r	(5) k_r	(6) $C_d = k_r C_r$	(7) H_r (m)
1.00	0.055	1.68	0.00	1.00	1.68	0.14
0.14	0.008	1.66	0.00	1.00	1.66	0.14

Column 1 is the assumed H_r in meters, Column 2 is H_r/L_r, Column 3 is C_r derived from H_r/L_r and H_r using Figure 7.6, Column 4 is y_d/H_r, Column 5 is k_r derived from y_d/H_r using Figure 7.6, Column 6 is C_d obtained by multiplying Columns 3 and 5 and Column 7 is obtained by substituting C_d in Column 6 into Equation 7.37 and solving for H_r. The iterations indicate that $C_d = 1.66$, $H_r = 0.14$ m, and the flow rate over the roadway, Q_r, is given by

$$Q_r = C_d L_R H_r^{\frac{3}{2}} = (1.66)(15.2)(0.14)^{\frac{3}{2}} = 1.32 \text{ m}^3/\text{s}$$

The corresponding flow rate through the culvert is equal to 2.49 m^3/s $-$ 1.32 m^3/s = 1.17 m^3/s. Therefore, a culvert diameter of 610 mm will result in roadway overtopping, with a flow rate of 1.17 m^3/s passing through the culvert, 1.32 m^3/s passing over the roadway, and a depth of flow over the roadway equal to 14 cm. A larger culvert diameter could be explored if less roadway overtopping at the design flow rate is desired.

7.2.5 Riprap/Outlet Protection

Outlet protection is required when there is a possibility of the native soil being eroded by the water exiting the culvert. This possibility is typically assessed by comparing the flow velocity at the culvert exit to the scour velocity of the native soil at the outlet. The velocity in the culvert barrel is assessed under design conditions and typical scour velocities of various soils are given in Table 7.6. In cases where the flow velocity at the culvert exit exceeds the scour velocity of the native soil, the native soil is usually overlain by riprap. *Riprap* consists of broken rock, cobbles, or boulders placed on the perimeter of a channel to protect against the erosive action of water. Riprap is a common erosion-control lining used at culvert outlets, storm-sewer outfalls, and around bridge abutments, especially in areas where suitable rock materials are readily available. A ground lining using riprap or any other material is commonly called an *apron*. A culvert exit with a riprap apron is shown in Figure 7.7. In this particular case, there is a small concrete apron between the culvert exit and the beginning of the riprap apron.

Design of riprap outlet protection includes specifying: (1) the type and size of stone, (2) the thickness of the stone lining, and (3) the length and width of the apron. Several design standards have been developed, and local regulatory requirements should always be followed if they exist. In lieu of regulatory requirements, the following design guidelines can be followed (Gribbin, 2007):

Type of Stone. Stones used for riprap should be hard, durable, and angular. Angularity, a feature of crushed stone from a quarry, helps to keep the stones locked together when subjected to the force of moving water.

TABLE 7.6: Scour Velocities of Various Soils

Soil	Permissible Velocity (m/s)	Permissible Velocity (ft/s)
Sand	0.5	1.6
Sandy loam	0.8	2.6
Silt loam	0.9	3.0
Sandy clay loam	1.1	3.6
Clay loam	1.2	3.9
Clay, fine gravel	1.5	4.9
Cobbles	1.7	5.6
Shale	1.8	5.9

FIGURE 7.7: Culvert exit with riprap apron