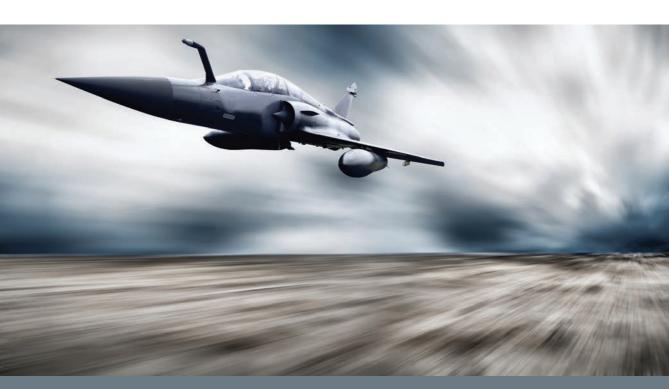


# Engineering Vibration

**FOURTH EDITION** 

Daniel J. Inman





# Fourth Edition

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steady state response assuming the system starts from rest. Also use the small angle approximation.

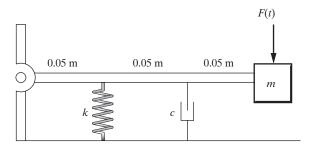


Figure P2.35

**2.36.** Consider the system of Problem 2.15, repeated here as Figure P2.36 with the effects of damping indicated. The physical constants are  $J=24\,\mathrm{kg}\,\mathrm{m}^2$ ,  $k=2500\,\mathrm{Nm/rad}$ , and the applied moment is 5 Nm at 1.432 Hz acting through the distance  $r=0.5\,\mathrm{m}$ . Compute the magnitude of the steady state response if the measured damping ratio of the spring system is  $\zeta=0.01$ . Compare this to the response for the case where the damping is not modeled ( $\zeta=0$ ).

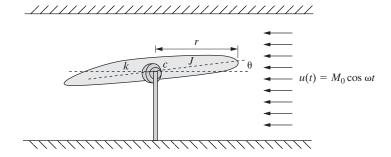


Figure P2.36 Model of an airfoil in a wind tunnel including the effects of damping.

- **2.37.** A machine, modeled as a linear spring-mass-damper system, is driven at resonance  $(\omega_n = \omega = 2 \text{ rad/s})$ . Design a damper (that is, choose a value of c) such that the maximum deflection at steady state is 0.05 m. The machine is modeled as having a stiffness of 2000 kg/m, and the excitation force has a magnitude of 100 N.
- **2.38.** Derive the total response of the system to initial conditions  $x_0$  and 0 using the homogenous solution in the form  $x_h(t) = e^{-\zeta \omega_n t} (A_1 \sin \omega_d t + A_2 \cos \omega_d t)$  and hence verify equation (2.38) for the forced response of an underdamped system.

### Section 2.3 (Problems 2.39 through 2.44)

**2.39.** Referring to Figure 2.11, draw the solution for the magnitude X for the case  $m=80\,\mathrm{kg}$ ,  $c=3200\,\mathrm{N}$  s/m, and  $k=8,000\,\mathrm{N/m}$ . Assume that the system is driven at resonance by a 10-N force.

- **2.40.** Use the graphical method to compute the phase shift for the system with m=100 kg, c=4000 N s/m,  $k=10{,}000$  N/m, and  $F_0=10$  N, if  $\omega=\omega_n/2$  and again for the case  $\omega=2\omega_n$ .
- 2.41. A body of mass 80 kg is suspended by a spring of stiffness of 25 kN/m and dashpot of damping constant 800 N s/m. Vibration is excited by a harmonic force of amplitude 60 N and a frequency of 3 Hz. Calculate the amplitude of the displacement for the vibration and the phase angle between the displacement and the excitation force using the graphical method.
- **2.42.** Calculate the real part of equation (2.55)

$$x_p(t) = \frac{F_0}{\left[\left(k - m\omega^2\right)^2 + (c\omega)^2\right]^{1/2}} e^{i(\omega t - \theta)}$$

to verify that this is consistent with the equation (2.36)

$$X_p = \frac{f_0}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}$$

and hence establish the equivalence of the exponential approach to solving the damped vibration problem with method of undetermined coefficients.

**2.43.** Referring to equation (2.56)

$$(ms^2 + cs + k)X(s) = \frac{F_0s}{s^2 + \omega^2}$$

and a table of Laplace transforms (see Appendix B), calculate the solution x(t) by using a table of Laplace transform pairs, and show that the solution obtained this way is equivalent to (2.36).

**2.44.** Solve the following system using the Laplace transform method and the table in Appendix B:

$$m\ddot{x}(t) + kx(t) = F_0 \cos \omega t, x(0) = x_0, \dot{x}(0) = v_0$$

Check your solution against equation (2.11) obtained via the method of undetermined coefficients.

## Section 2.4 (Problems 2.45 through 2.60)

**2.45.** For a base motion system described by

$$m\ddot{x} + c\dot{x} + kx = cY\omega_b\cos\omega_b t + kY\sin\omega_b t$$

with m = 100 kg, c = 50 kg/s, k = 1000 N/m, Y = 0.03 m, and  $\omega_b = 3$  rad/s, compute the magnitude of the particular solution. Last, compute the transmissibility ratio.

**2.46.** For a base motion system described by

$$m\ddot{x} + c\dot{x} + kx = cY\omega_b\cos\omega_b t + kY\sin\omega_b t$$

with m = 100 kg, c = 50 N/m, Y = 0.03 m, and  $\omega_b = 3$  rad/s, find largest value of the stiffness k and that makes the transmissibility ratio less than 0.75.

**2.47.** A machine weighing 1800 N rests on a support as illustrated in Figure P2.47. The support deflects about 4 cm as a result of the weight of the machine. The floor under the support is somewhat flexible and moves, because of the motion of a nearby machine, harmonically near resonance (r=1) with an amplitude of 0.2 cm. Model the floor as base motion, and assume a damping ratio of  $\zeta = 0.01$ , and calculate the transmitted force and the amplitude of the transmitted displacement.

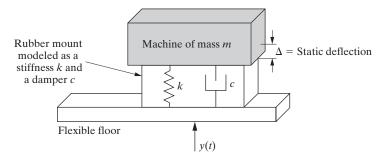


Figure P2.47

**2.48.** Derive equation (2.70)

$$X = Y \left[ \frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

from (2.68)

$$x_p(t) = \omega_n Y \left[ \frac{\omega_n^2 + (2\zeta\omega_b)^2}{(\omega_n^2 - \omega_b^2)^2 + (2\zeta\omega_n\omega_b)^2} \right]^{1/2} \cos(\omega_b t - \theta_1 - \theta_2)$$

to see if the author has done it correctly.

- **2.49.** From the equation describing Figure 2.14, show that the point  $(\sqrt{2}, 1)$  corresponds to the value TR > 1 (i.e., for all  $r < \sqrt{2}, TR > 1$ ).
- **2.50.** Consider the base-excitation problem for the configuration shown in Figure P2.50. In this case, the base motion is a displacement transmitted through a dashpot or pure damping element. Derive an expression for the force transmitted to the support in steady state.

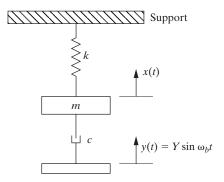
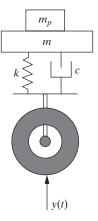


Figure P2.50

**2.51.** A very common example of base motion is the single-degree-of-freedom model of an automobile driving over a rough road. The road is modeled as providing a base motion displacement of  $y(t) = (0.01)\sin(5.818t)$  m. The suspension provides an equivalent stiffness of  $k = 3.273 \times 10^4$  N/m, a damping coefficient of c = 231 kg/s, and a mass of 1007 kg. Determine the amplitude of the absolute displacement of the automobile mass.

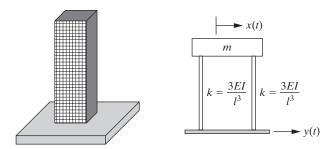
- 2.52. A vibrating mass of 250 kg, mounted on a massless support by a spring of stiffness 32,000 N/m and a damper of unknown damping coefficient, is observed to vibrate with a 10-mm amplitude while the support vibration has a maximum amplitude of only 2.5 mm (at resonance). Calculate the damping constant and the amplitude of the force on the base.
- **2.53.** Referring to Example 2.4.2, at what speed does Car 1 experience resonance? At what speed does Car 2 experience resonance? Calculate the maximum deflection of both cars at resonance.
- **2.54.** For cars of Example 2.4.2, calculate the best choice of the damping coefficient so that the transmissibility is as small as possible by comparing the magnitude of  $\zeta = 0.01$ ,  $\zeta = 0.1$ , and  $\zeta = 0.2$  for the case r = 2. What happens if the road "frequency" changes?
- **2.55.** A system modeled by Figure 2.13, has a mass of 200 kg with a spring stiffness of  $3.0 \times 10^4 \,\mathrm{N/m}$ . Calculate the damping coefficient given that the system has a deflection (X) of 0.7 cm when driven at its natural frequency while the base amplitude (Y) is measured to be 0.3 cm.
- 2.56. Consider Example 2.4.2 for Car 1 illustrated in Figure P2.56 if three passengers totaling 200 kg are riding in the car. Calculate the effect of the mass of the passengers on the deflection at 20, 80, 100, and 150 km/h. What is the effect of the added passenger mass on Car 2?



**Figure P2.56** A model of a car suspension with the mass of the occupants,  $m_p$ , included.

**2.57.** Consider Example 2.4.2. Choose values of *c* and *k* for the suspension system for Car 2 (the sedan) such that the amplitude transmitted to the passenger compartment is as small as possible for the 1-cm bump at 50 km/h. Also calculate the deflection at 100 km/h for your values of *c* and *k*.

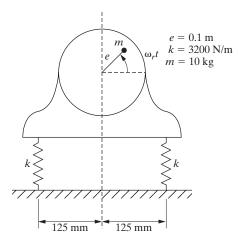
- **2.58.** Consider the base motion problem of Figure 2.13. (a) Compute the damping ratio needed to keep the displacement magnitude transmissibility less then 0.50 for a frequency ratio of r = 1.5. (b) What is the value of the force transmissibility ratio for this system?
- **2.59.** Consider the effect of variable mass on an aircraft landing suspension system by modeling the landing gear as a moving base problem similar to that shown in Figure P2.56 for a car suspension. The mass of a regional jet is 13,236 kg empty and its maximum takeoff mass is 21,523 kg. Compare the maximum deflection for a wheel motion of magnitude 0.50 m and frequency of 35 rad/s for these two different masses. Take the damping ratio to be  $\zeta = 0.1$  and the stiffness to be  $4.22 \times 10^6$  N/m.
- 2.60. Consider the simple model of a building subject to ground motion suggested in Figure P2.60. The building is modeled as a single-degree-of-freedom spring-mass system where the building mass is lumped atop two beams used to model the walls of the building in bending. Assume the ground motion is modeled as having amplitude of 0.1 m at a frequency of 7.5 rad/s. Approximate the building mass by 10<sup>5</sup> kg and the stiffness of each wall by 3.519 × 10<sup>6</sup> N/m. Compute the magnitude of the deflection of the top of the building.



**Figure P2.60** A simple model of a building subject to ground motion, such as an earthquake.

### Section 2.5 (Problems 2.61 through 2.68)

- **2.61.** A lathe can be modeled as an electric motor mounted on a steel table. The table plus the motor have a mass of 60 kg. The rotating parts of the lathe have a mass of 4 kg at a distance 0.12 m from the center. The damping ratio of the system is measured to be  $\zeta = 0.06$  (viscous damping) and its natural frequency is 7.5 Hz. Calculate the amplitude of the steady-state displacement of the motor, assuming  $\omega_r = 30 \, \text{Hz}$ .
- **2.62.** The system of Figure 2.19 produces a forced oscillation of varying frequency. As the frequency is changed, it is noted that at resonance the amplitude of the displacement is 10 mm. As the frequency is increased several decades past resonance, the amplitude of the displacement remains fixed at 1 mm. Estimate the damping ratio for the system.
- **2.63.** An electric motor (Figure P2.63) has an eccentric mass of 12 kg (12% of the total mass of 100 kg) and is set on two identical springs ( $k = 3000 \,\mathrm{N/m}$ ). The motor runs at 1800 rpm, and the mass eccentricity is 100 mm from the center. The springs are mounted 250 mm apart with the motor shaft in the center. Neglect damping and determine the amplitude of the vertical vibration.



**Figure P2.63** A vibration model for an electric motor with an unbalance.

- **2.64.** Consider a system with rotating unbalance as illustrated in Figure P2.63. Suppose the deflection at 1800 rpm is measured to be 0.05 m and the damping ratio is measured to be  $\zeta = 0.1$ . The out-of-balance mass is estimated to be 10%. Locate the unbalanced mass by computing e.
- **2.65.** A fan of 45 kg has an unbalance that creates a harmonic force. A spring-damper system is designed to minimize the force transmitted to the base of the fan. A damper is used having a damping ratio of  $\zeta = 0.2$ . Calculate the required spring stiffness so that only 10% of the force is transmitted to the ground when the fan is running at 10,000 rpm.
- **2.66.** Plot the normalized displacement magnitude versus the frequency ratio for the out-of-balance problem (i.e., repeat Figure 2.21) for the case of  $\zeta = 0.05$ .
- **2.67.** Consider a typical unbalanced machine problem as given in Figure P2.67 with a machine mass of 150 kg, a mount stiffness of 1000 kN/m and a damping value of 600 kg/s.

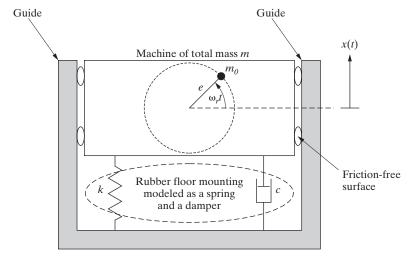


Figure P2.67 A typical unbalance machine problem.