INTERNATIONAL EDITION

Optoelectronics and Photonics

Principles and Practices

SECOND EDITION

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ALWAYS LEARNING

PEARSON

Selected definitions and basic equations

Photon energy

$$E_{ph} = h\upsilon = \hbar\omega;\ \omega = 2\pi\upsilon$$

Photon momentum

$$p_{ph} = \frac{h}{\lambda} = \hbar k$$

Photon flux Φ_{ph} and irradiance (intensity)

$$\Phi_{ph} = \frac{\text{Photons crossing area } A \text{ in time } \Delta t}{A\Delta t} = \frac{\Delta N_{ph}}{A\Delta t}$$

$$I = h\upsilon\Phi_{ph}$$

Propagation constant (wave vector)

$$k = \frac{2\pi}{\lambda}$$

Phase velocity

$$v = \lambda\upsilon = \frac{\omega}{k};\quad v = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r}}$$

Changes in wavelength and frequency

$$\frac{\delta\lambda}{\lambda} = -\frac{\delta\upsilon}{\upsilon};\quad \delta\lambda = -\frac{\lambda^2}{c}\delta\upsilon = -\frac{c}{\upsilon^2}\delta\upsilon$$

Group velocity

$$v_g = \frac{d\omega}{dk}$$

Group index

$$v_g(\text{medium}) = \frac{c}{N_g};\quad N_g = n - \lambda_o\frac{dn}{d\lambda_o}$$

Electric and magnetic fields

$$E_x = vB_y = \frac{c}{n}B_y$$

Poynting vector and irradiance

$$\mathbf{S} = v^2\varepsilon_o\varepsilon_r\mathbf{E}\times\mathbf{B};\quad I = S_{\text{average}} = \frac{1}{2}v\varepsilon_o\varepsilon_r E_o^2$$

Snell's law and the Brewster angle

$$n_1\sin\theta_i = n_2\sin\theta_t; \sin\theta_c = \frac{n_2}{n_1}; \tan\theta_p = \frac{n_2}{n_1}$$

Phase change in total internal reflection (TIR)

$$\tan\left(\tfrac{1}{2}\phi_\perp\right) = \frac{[\sin^2\theta_i - n^2]^{1/2}}{\cos\theta_i};\ n = \frac{n_2}{n_1}$$

$$\tan\left(\tfrac{1}{2}(\phi_{//} + \pi)\right) = \frac{[\sin^2\theta_i - n^2]^{1/2}}{n^2\cos\theta_i}$$

Attenuation in second medium in TIR

$$\alpha_2 = \frac{2\pi n_2}{\lambda_o}\left[\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1\right]^{1/2}$$

Reflectance, transmittance (normal incidence)

$$R = R_\perp = R_{//} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2;$$

$$T = T_\perp = T_{//} = \frac{4n_1n_2}{(n_1 + n_2)^2}$$

Fabry–Perot cavity

$$\upsilon_m = m\left(\frac{c}{2L}\right) = m\upsilon_f,\quad m = 1, 2, 3, \ldots$$

$$\delta\upsilon_m = \frac{\upsilon_f}{F};\quad F = \frac{\pi R^{1/2}}{1 - R}$$

Single slit diffraction

$$I(\theta) = I(0)\text{sinc}^2(\beta);\quad \beta = \frac{1}{2}(ka\sin\theta)$$

Airy disk, angular radius, divergence

$$\sin\theta_o = 1.22\frac{\lambda}{D}$$

$$\text{Divergence} = 2\theta_o \approx 2\times 1.22\frac{\lambda}{D}$$

Diffraction grating

$$d(\sin\theta_m - \sin\theta_i) = m\lambda;\quad m = 0, \pm1, \pm2, \ldots$$

V-number, normalized frequency

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2};\ V = \frac{2\pi a}{\lambda}\text{NA}$$

Normalized index difference

$$\Delta = (n_1 - n_2)/n_1$$

Acceptance angle and numerical aperture (NA)

$$2\alpha_{\max}; \sin\alpha_{\max} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_0}; \sin\alpha_{\max} = \frac{\text{NA}}{n_0}$$

Normalized propagation constant

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{(\beta/k) - n_2}{n_1 - n_2}$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \text{ for } 1.5 < V < 2.5$$

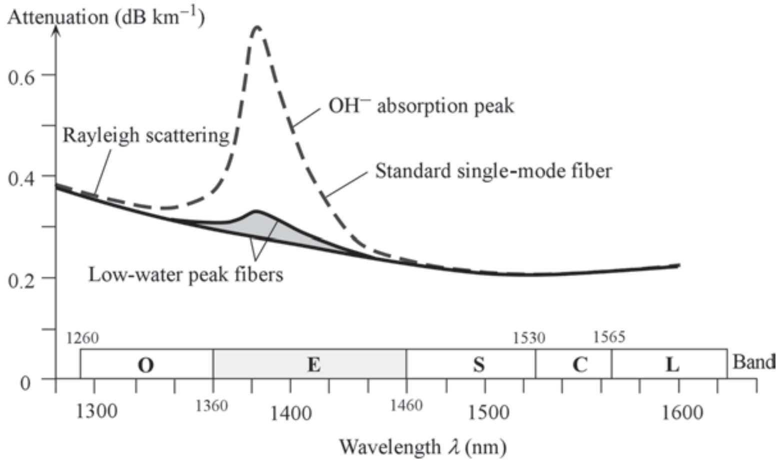

FIGURE 2.35 Typical attenuation vs. wavelength for a standard single-mode optical fiber and also in a low water peak fiber. These fibers are called *all-wave fiber.*

By proper design, the attenuation window at around 1.5 μm may be lowered to approach the Rayleigh scattering limit. The OH$^-$ peak at around 1.38 μm has prevented the use of the E-band in optical communications around this wavelength. However, recent developments in fiber manufacturing have almost totally eliminated the water peak as apparent in the α vs. λ characteristic of a newly developed low water peak (LWP) fiber shown in Figure 2.35. LWP fibers are commercially available with the attenuation being near the scattering limit from 1290 nm to 1625 nm. The ITU-T G.652D standard defines what we expect from commercial LWP fibers.

C. Intrinsic Attenuation Equations

The tail of the *fundamental infrared absorption* can be represented by a simple exponential

Fundamental lattice absorption

$$\alpha_{\text{FIR}} = A\exp\left(-\frac{B}{\lambda}\right) \tag{2.9.7}$$

where for silica fibers $A = 7.81 \times 10^{11}$ dB km^{-1}, $B = 48.5$ μm. Although A may seem unreasonably large, Eq. (2.9.7) predicts the right values in the tail region of lattice absorption. These values depend on the composition of the core.

Rayleigh scattering represents the lowest attenuation one can achieve using a glass structure. As mentioned above, the Rayleigh scattering process decreases with wavelength, and it is inversely proportional to λ^4. The attenuation α_R of a silica fiber due to Rayleigh scattering can be written as

Rayleigh scattering in silica

$$\alpha_R = \frac{A_R}{\lambda^4} \tag{2.9.8}$$

in which λ is the free-space wavelength and A_R is a coefficient that depends on the composition of the core. It also depends on the thermal history involved in manufacturing the fiber. Many books quote $A_R \approx 0.90$ dB km^{-1} for estimating the Rayleigh scattering loss. As shown in Example 2.9.2, it serves as a good rule of thumb, given that it contains no information on the core composition or the refractive index difference. For pure silica (undoped) preforms from which fibers are drawn, A_R is smaller and about 0.63 dB km^{-1}. This attenuation increases as dopants are introduced.

TABLE 2.6 Approximate attenuation coefficients for silica-based fibers for use in Eqs. (2.9.7) and (2.9.8)

Glass	A_R(dB km^{-1})	Comment
Silica fiber	0.90	"Rule of thumb"
SiO_2-GeO_2 core step-index fiber	$0.63 + 2.06 \times \text{NA}$	NA depends on $(n_1^2 - n_2^2)^{1/2}$ and hence on the doping difference.
SiO_2-GeO_2 core graded index fiber	$0.63 + 1.75 \times \text{NA}$	
Silica, SiO_2	0.63	Measured on preforms. Depends on annealing. $A_R(\text{Silica}) = 0.59\,\text{dB km}^{-1}$ for annealed.
65%$SiO_2$35%GeO_2	0.75	On a preform. $A_R/A_R(\text{silica}) = 1.19$
$(SiO_2)_{1-x}(GeO_2)_x$	$A_R(\text{silica}) \times (1 + 0.62x)$	$x = [GeO_2]$ = Concentration as a fraction (10% GeO_2, $x = 0.1$). For preform.

(*Source:* Data mainly from K. Tsujikawa *et al.*, *Electron. Letts.*, *30*, 351, 1994; *Opt. Fib. Technol.*, *11*, 319, 2005; H. Hughes, *Telecommunications Cables*, John Wiley & Sons, 1999, and references therein.)

Note: Square brackets represent concentration as a fraction. NA = numerical aperture. $A_R = 0.59$ used as reference for pure silica, and represents A_R(silica).

Table 2.6 summarizes some typical values for the Rayleigh scattering coefficient, A_R in Eq. (2.9.8). A_R increases as silica is doped, for example, with GeO_2 (germania), to modify its refractive index, and the increase depends on the dopant concentration. Further, given one type of dopant, for example, GeO_2, the refractive index change is approximately proportional to the concentration of dopants so that A_R can also be expressed in terms of the numerical aperture (NA). The lowest attenuation at 1.55 μm for a silica fiber has been reported to be about 0.15 dB km^{-1}.

EXAMPLE 2.9.2 Rayleigh scattering equations

Consider a single-mode step-index fiber, which has a numerical aperture of 0.14. Predict the expected attenuation at 1.55 μm, and compare your calculation with the reported (measured) value of 0.19–0.20 dB km^{-1} for this fiber. Repeat the calculations at 1.31 μm, and compare your values with the reported 0.33–0.35 dB km^{-1} values.

Solution

First, we should check the fundamental infrared absorption at 1550 nm. Using Eq. (2.9.7)

$$\alpha_{\text{FIR}} = A\exp(-B/\lambda) = 7.8 \times 10^{11}\exp\left[-(48.5)/(1.55)\right] = 0.020\,\text{dB km}^{-1}$$

which is very small.

Next, for the Rayleigh scattering at 1550 nm, the simplest equation with $A_R = 0.90$ dB km^{-1} μm^4, gives

$$\alpha_R = A_R/\lambda^4 = (0.90\,\text{dB km}^{-1}\,\mu\text{m}^4)/(1.55\,\mu\text{m})^4 = 0.156\text{ dB km}^{-1}$$

This equation is basically a rule of thumb. The total attenuation is then

$$\alpha_R + \alpha_{\text{FIR}} = 0.156 + 0.02 = 0.176\,\text{dB km}^{-1}$$

The above "rule of thumb" predicted value is very close to that reported for the Corning SMG28e$^+$ SMF.

The current fiber has NA $= 0.14$. Thus

$$A_R = 0.63 + 2.06 \times \text{NA} = 0.63 + 2.06 \times 0.14 = 0.918\,\text{dB km}^{-1}\,\mu\text{m}^4$$

i.e.,

$$\alpha_R = A_R/\lambda^4 = (0.918\,\text{dB km}^{-1}\,\mu\text{m}^4)/(1.55\,\mu\text{m})^4 = 0.159\text{ dB km}^{-1}$$

which gives a total attenuation of $0.159 + 0.020$ or 0.179 dB km^{-1}.

We can repeat the above calculations at $\lambda = 1.31\,\mu$m. However, we do not need to add α_{FIR}.

$$\alpha_R = A_R/\lambda^4 = (0.90\text{ dB km}^{-1}\,\mu\text{m}^4)/(1.31\,\mu\text{m})^4 = 0.306\text{ dB km}^{-1}$$

and using the NA-based value for A_R from above

$$\alpha_R = A_R/\lambda^4 = (0.918\,\text{dB km}^{-1}\,\mu\text{m}^4)/(1.31\,\mu\text{m})^4 = 0.312\text{ dB km}^{-1}$$

Both close to the measured value.

D. Bending losses

External factors, such as bending, can also lead to attenuation in the optical fiber. **Bending losses** are those losses that arise whenever a fiber is bent as illustrated in Figure 2.36. Bending losses are classified into two broad categories: microbending and macrobending losses, which are illustrated in Figure 2.36. **Microbending loss** occurs whenever the radius of curvature of the bend is sharp, that is, when the bend radius is comparable to the diameter of the fiber as shown in Figure 2.36 (a). Typically microbending losses are significant when the radius of curvature of the bend is less than 0.1–1 mm. They can arise from careless or poor cabling of the fiber or even from flaws in manufacturing that result in variations in the fiber geometry over small distances. **Macrobending losses** are those losses that arise when the bend is much larger than the fiber size, typically much greater than 1 mm, as illustrated in Figure 2.36 (b). They typically occur when the fiber is bent during the installation of a fiber optic link such as turning the fiber around a corner. There is no simple precise and sharp boundary line between microbending and macrobending loss definitions. Both losses essentially result from changes in the waveguide geometry and properties as the fiber is subjected to external forces that bend the fiber. Typically, macrobending loss crosses over into microbending loss when the radius of curvature becomes less than a few millimeters. Below we simply use the term *bending loss* to describe those significant losses arising from the bending of the fiber.

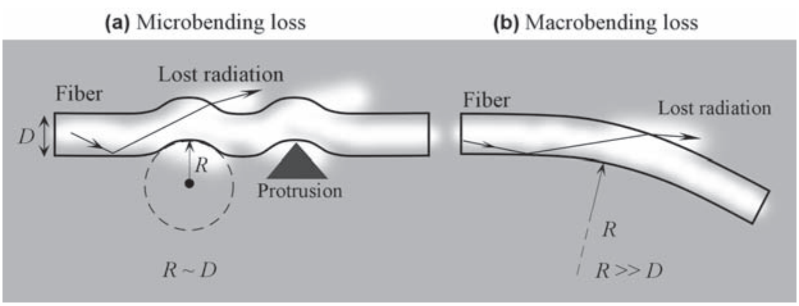

FIGURE 2.36 Definitions of (a) microbending and (b) macrobending loss and the definition of the radius of curvature, *R*. (A schematic illustration only.) The propagating mode in the fiber is shown as white-shaded area. Some radiation is lost in the region where the fiber is bent. *D* is the fiber diameter, including the cladding.

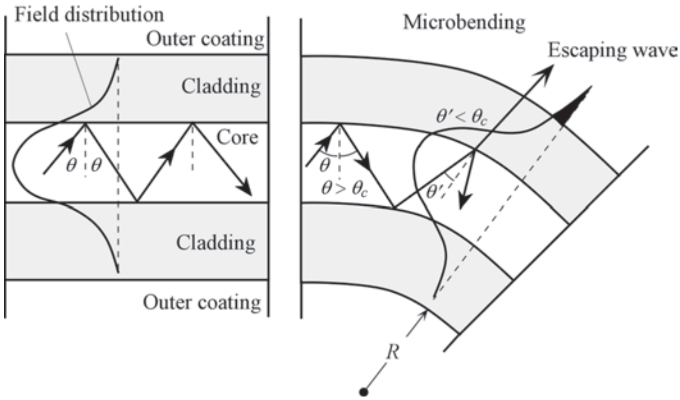

FIGURE 2.37 Sharp bends change the local waveguide geometry that can lead to waves escaping. The zigzagging ray suddenly finds itself with an incidence angle smaller than θ' that gives rise to either a transmitted wave, or to a greater cladding penetration; the field reaches the outside medium and some light energy is lost.

When a fiber is bent, the guide geometry and the refractive index profile change locally, which leads to some of the light energy radiating away from the guiding direction. A sharp bend, as illustrated intuitively in Figure 2.37, will change the local waveguide geometry in such a way that a zigzagging ray suddenly finds itself with an incidence angle θ', narrower than its normal angle $\theta(\theta' < \theta)$, which gives rise to either a transmitted wave (a refracted wave into the cladding) or to a greater cladding penetration. If $\theta' < \theta_c$, the critical angle (which would have been modified by the bent), then there will be no total internal reflection and substantial light power will be radiated into the cladding where it will be lost, by, for example, reaching the outer coating (polymer coating, *etc.*) where losses are significant. Attenuation increases sharply with the extent of bending; as θ' gets narrow and TIR is lost, substantially more energy is transferred into the cladding. Further, highest modes propagate with incidence angles θ close to θ_c, which means that these modes are most severely affected. Multimode fibers therefore suffer more from bending losses than single-mode fibers.

While Figure 2.37 provides an intuitive explanation, it is important to understand bending losses in terms of the mode field diameter of the fiber and what happens to the evanescent wave in the cladding upon bending. When a fiber is bent sharply, the propagating wavefront along the straight fiber cannot bend around and continue as a wavefront because a portion of it beyond some critical radial distance r_c, as shown in Figure 2.38, has to travel a *longer* distance and to

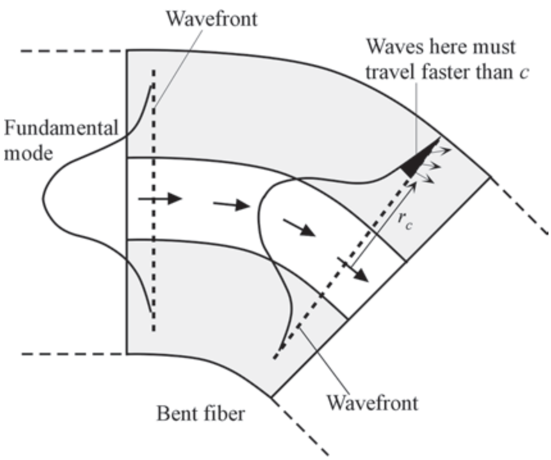

FIGURE 2.38 When a fiber is bent sharply, the propagating wavefront along the straight fiber cannot bend around and continue as a wavefront because a portion of it (black shaded) beyond the critical radial distance r_c must travel faster than the speed of light in vacuum. This portion is lost in the cladding, that is, radiated away.

stay in phase it must travel faster than the speed of light in vacuum, *c*. Since this is impossible, this portion is lost in the cladding, *i.e.*, radiated away from the propagating mode, which reduces the power in it.

The bending loss α_B increases rapidly with increasing bend sharpness, that is, with decreasing radius of bend curvature, *R*. Figure 2.39 (a) shows how the bending loss α_B depends on the radius of curvature *R* for three types of single-mode fibers: a standard SMF and two trench fibers. Trench fibers have the refractive index profile as shown in Figure 2.39 (b). There is a refractive index trench placed in the cladding where the refractive index n_3 is lower than the cladding index (n_2). Note also that the outer cladding, after the trench, has a different refractive index, n_4. The confinement of the light wave in trench fibers is such that they are relatively insensitive to bending compared to standard fibers. As apparent, α_B increases exponentially with decreasing *R* as shown in Figure 2.39 (a), which is a semilogarithmic plot, that is,

Bending loss

$$\alpha_B = A\exp(-R/R_c) \qquad \textbf{(2.9.9)}$$

where *A* and R_c are constants that depend on the fiber properties. Note that α_B is normally reported in dB/turn of fiber for a given radius of curvature as in Figure 2.39 (a).

For single-mode fibers, a quantity called a **MAC-value**, or a **MAC-number**, N_{MAC}, has been used to characterize the bending loss. N_{MAC} is defined by

MAC-number

$$N_{\text{MAC}} = \frac{\text{Mode field diameter}}{\text{Cutoff wavelength}} = \frac{\text{MFD}}{\lambda_c} \qquad \textbf{(2.9.10)}$$

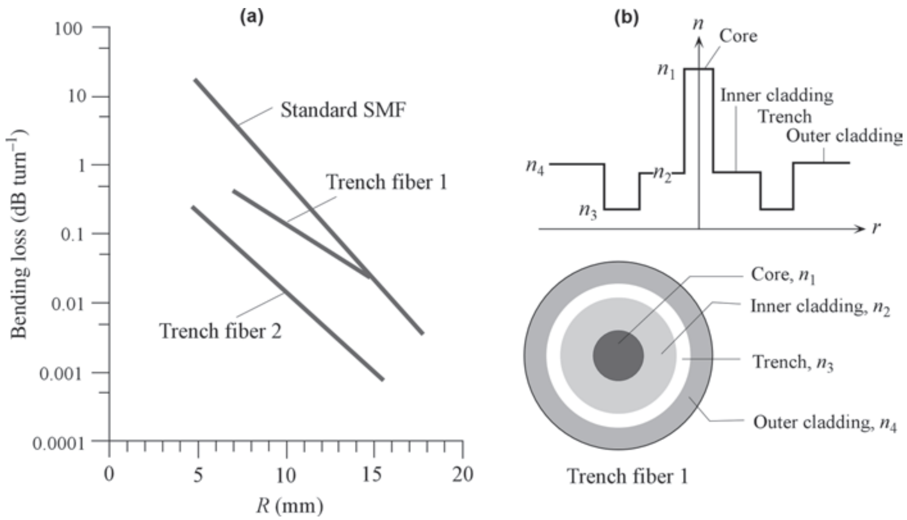

FIGURE 2.39 (a) Bending loss in dB per turn of fiber for three types of fibers: standard single mode and two trench fibers, around 1.55–1.65 μm. (b) The index profile for the trench fiber 1 in (a), and a schematic view of the fiber cross-section. Experimental data used to generate the plots have been combined from various sources. (*Source:* Standard fiber, M.-J. Li *et al.*, *J. Light Wave Technol.*, *27*, 376, 2009; Trench fiber 1, K. Himeno *et al.*, *J. Light Wave Technol.*, *23*, 3494, 2005, Trench fiber 2, L.-A. de Montmorillon *et al.*, "Bend-Optimized G.652D Compatible Trench-Assisted Single-Mode Fibers," *Proceedings of the 55th IWCS/Focus*, pp. 342–347, November, 2006.)

The above definition is based on observations that α_B generally increases with increasing MFD and decreasing cutoff wavelength. The bending loss increases exponentially with the MAC-number. It is not a universal function, and different types of fibers, with very different refractive index profiles, can exhibit different α_B vs. N_{MAC} behavior. N_{MAC} values are typically in the range 6–8.

It is clear from Figure 2.38 that bending losses involve the escape of power from the propagating mode when the waves beyond the critical distance r_c in the cladding cannot keep up with the overall propagation of the mode. The extent of penetration into the cladding is therefore important. It should, in principle, be possible to shape the refractive index profile in the "cladding" to better contain the radiation to the core, and hence avoid losses. Indeed, bending losses can be reduced by designing a suitable cladding that constrains the power in the propagating mode along the core even when the fiber is bent. The simple step-index fiber cannot constrain the power to stay within the core when the fiber is sufficiently bent. The trench fibers shown in Figure 2.39 (b) are able to reduce the bending loss by ensuring that the fiber has to be bent much more sharply than the standard fiber for the field to reach r_c. Thus, the optical power is better confined within the core region.

Bending loss is of particular interest in MMFs. Since MMFs are used in short-haul networks such as links within buildings, they invariably become bent in installation. Technicians would like to bend a fiber just like a copper or a coaxial cable in laying down connections. Bending losses of a fiber are typically measured by wounding the fiber for N turns on a drum or a mandrel of certain radius R, and then measuring the attenuation of the fiber. The standard OM1 MMF for use at 850 nm and 1310 nm optical links has been specified to have a maximum bending (macrobending) loss of 0.5 dB when the fiber is wound 100 turns with a radius 75 mm, *i.e.*, a bending loss of 0.005 dB turn^{-1} for a bend radius of 75 mm. Bend-insensitive fibers have been designed to have lower bend losses. For example, some fiber manufacturers specify the allowed bend radius for a given level of attenuation at a certain wavelength (*e.g.*, 1310 nm).

EXAMPLE 2.9.3 Bending loss for SMF

Experiments on a standard SMF operating around 1550 nm have shown that the bending loss is 0.124 dB turn^{-1} when the bend radius is 12.5 mm and 15.0 dB turn^{-1} when the bend radius is 5.0 mm. What is the loss at a bend radius of 10 mm?

Solution

We apply Eq. (2.9.9), $\alpha_B = A\exp(-R/R_c)$, for the two set of values given,

$$0.124 \text{ dB turn}^{-1} = A\exp\left[-(12.5\,\text{mm})/R_c\right] \quad \text{and} \quad 15.0 \text{ dB turn}^{-1} = A\exp\left[-(5.0\,\text{mm})/R_c\right]$$

We have two equations, and two unknowns, A and R_c. Dividing the first by the second and separating out R_c we find

$$R_c = (12.5 \text{ mm} - 5.0 \text{ mm})/\ln(15.0/0.124) = 1.56 \text{ mm}$$

and

$$A = (15.0\,\text{dB turn}^{-1})\exp\left[(5.0\,\text{mm})/(1.56\,\text{mm})\right] = 370 \text{ dB turn}^{-1}$$

Thus, at $R = 10\,\text{mm}$, $\alpha_B = A\exp(-R/R_c) = (370)\exp\left[-(10\,\text{mm})/(1.56\,\text{mm})\right] = 0.61\,\text{dB turn}^{-1}$.

The experimental value is also 0.61 dB turn^{-1} to within two decimal places.