Fifth Edition

Quantitative Methods for Business

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Pearson

Quantitative Methods for Business

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And because the base value is constant, you can take any other period, m, and say that:

$$\frac{\text{base value}}{100} = \frac{\text{value in period } n}{\text{index for period } n} = \frac{\text{value in period } m}{\text{index for period } m}$$

This is how you compare values at different times, as illustrated in Worked example 7.3.

WORKED EXAMPLE 7.3

The table shows the monthly index for sales of an item.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Index	121	112	98	81	63	57	89	109	131	147	132	126

(a) If sales in month 3 are 240 units, what are sales in month 8?
(b) If sales in month 10 are 1,200 units, what are sales in month 2?

Solution

(a) Using the ratios:

$$\frac{\text{sales in month 8}}{\text{index in month 8}} = \frac{\text{sales in month 3}}{\text{index in month 3}}$$

$$\frac{\text{sales in month 8}}{109} = \frac{240}{98}$$

or

$$\text{sales in month 8} = 240 \quad 109 / 98$$
$$= 267$$

(b) Again you can use the indices directly to give:

$$\frac{\text{sales in month 2}}{\text{index in month 2}} = \frac{\text{sales in month 10}}{\text{index in month 10}}$$

or

$$\text{sales in month 2} = 1{,}200 \quad 112 / 147 = 914$$

We have described a standard format for indices, but remember that:

- You can use an index to measure the way that any variable – not just price – changes over time.
- The usual base value is 100, but this is only for convenience and you can use any other value.
- You can choose the base period as any appropriate point for comparisons. It is usually a typical period with no unusual circumstances – or it might be a period that you are particularly interested in, such as the first period of a financial review.
- You can calculate an index with any convenient frequency, such as monthly indices for unemployment, daily indices for stock market prices, quarterly indices for production and annual indices for GNP.

Review questions

7.1 What is the purpose of an index?

7.2 Indices always use a base value of 100. Why is this?

7.3 What is the difference between a rise of 10% and a rise of 10 percentage points?

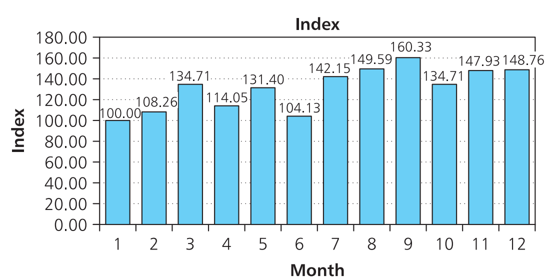
IDEAS IN PRACTICE Mohan Dass and Partners

In 2006 Mohan Dass bought out the other partners in a company that distributes medical supplies around the Middle East. He immediately started a programme of improvement and hopes to see the results during the period 2008 to 2013. In particular, he wants the company to expand rapidly, with turnover increasing by 100% a year for the next five years. To achieve this he is focusing on sales through the company website, introducing generic brands, improving logistics flows, expanding the product range, moving into new geographical areas, forming partnerships with major suppliers and customers and raising the company profile with health service providers.

To monitor his progress, Mohan collects information about operations, illustrated in Figure 7.2 which shows the index of sales over the past year. Mohan continually monitors a set of 82 measures of this kind to show different aspects of company performance.

Indices for Mohan Dass and Partners

Month	Sales	Index	Percentage change	Percentage point change
1	12.1	100.00	0	0
2	13.1	108.26	8.26	8.26
3	16.3	134.71	24.43	26.45
4	13.8	114.05	–15.34	–20.66
5	15.9	131.40	15.22	17.36
6	12.6	104.13	–20.75	–27.27
7	17.2	142.15	36.51	38.02
8	18.1	149.59	5.23	7.44
9	19.4	160.33	7.18	10.74
10	16.3	134.71	–15.98	–25.62
11	17.9	147.93	9.82	13.22
12	18	148.76	0.56	0.83



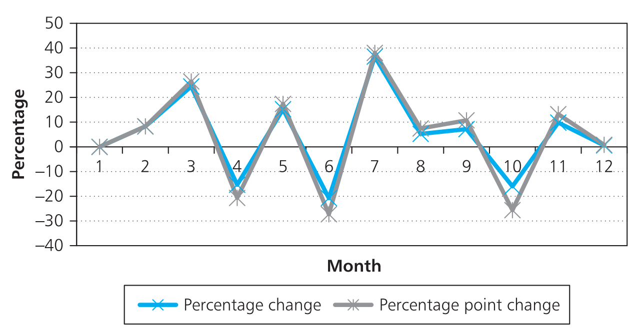


Figure 7.2 Index of sales at Mohan Dass and Partners

Source: Richmond E., Internal Report 147/06, Richmond, Parkes and Wright, Cairo, 2006.

Changing the base period

An index can use any convenient base period, but rather than keep the same one for a long time it is often best to update it periodically. There are two reasons for this:

- *Changing circumstances* – you should reset an index whenever there are significant changes that make comparisons with earlier periods meaningless. For example, a service provider might use an index to monitor the number of customers, but should change the base year whenever there are significant changes to the service offered.
- *An index becomes too large* – when an index rises to, say, 5,000 a 10% increase raises it by 500 points, and this seems a much more significant change than a jump from 100 to 110.

On the other hand, changing the base period introduces a discontinuity that makes comparisons over long periods more difficult. This is why people often keep the same base even when it becomes very high (like the Nikkei index of the Tokyo stock exchange which was once approaching 20,000).

In practice, it is easy to convert between indices. When you have an old index that is calculated from an old base value, the old index for period M is:

$$\text{old index} = \frac{\text{value in period } M}{\text{old base value}} \quad 100$$

or

$$\text{old index} \quad \text{old base value} = \text{value in period } M \quad 100$$

Now calculating a new index for period M using a new base period gives:

$$\text{new index} = \frac{\text{value in period } M}{\text{new base value}} \quad 100$$

or

$$\text{new index} \quad \text{new base value} = \text{value in period } M \quad 100$$

These two equations are clearly the same, so we can write:

$$\text{old index} \quad \text{old base value} = \text{new index} \quad \text{new base value}$$

or

$$\text{new index} = \text{old index} \quad \frac{\text{old base value}}{\text{new base value}}$$

As both the old and new base values are fixed, you find the new index by multiplying the old index by a constant. For example, if the old base value was 200 and the new base value is 500, you always find the new index for any period by multiplying the old index by 200 / 500.

WORKED EXAMPLE 7.4

The following indices monitor the annual profits of J.R. Hartman and Associates.

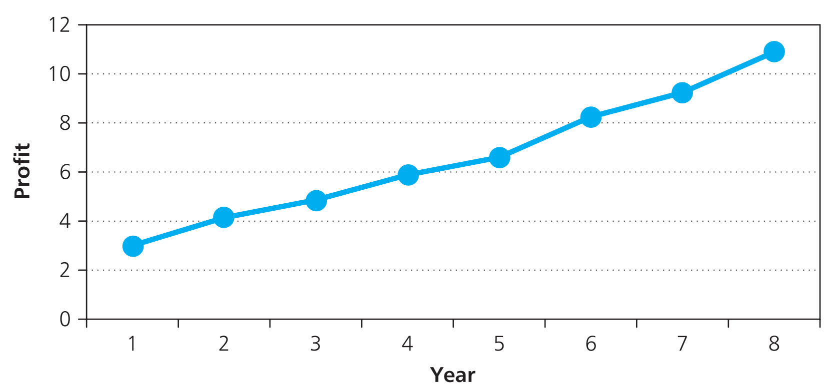
Year	1	2	3	4	5	6	7	8
Index 1	100	138	162	196	220			
Index 2					100	125	140	165

(a) What are the base years for the indices?
(b) If the company had not changed to Index 2, what values would Index 1 have in years 6 to 8?
(c) What values does Index 2 have in years 1 to 4?
(d) If the company made a profit of €4.86 million in year 3, how much did it make in the other years?

Solution

(a) Indices generally have a value of 100 in base periods, so Index 1 uses the base year 1 and Index 2 uses the base year 5.
(b) You find Index 1 by multiplying Index 2 by a constant amount. You can find this constant from year 5, when Index 1 is 220 and Index 2 is 100 – so to convert to Index 1 from Index 2 you multiply by 220 / 100. Then Index 1 for year 6 is 125 220 / 100 = 275, and so on, as shown in Figure 7.3.
(c) Using the same reasoning, you change to Index 2 from Index 1 by multiplying by 100 / 220. Index 2 for year 4 is 196 100 / 220 = 89.09 and so on, as shown in Figure 7.3.

	A	B	C	D	E	F	G	H	I
1	**Indices for profit at J.R. Hartman and Associates**								
2									
3	**Year**	1	2	3	4	5	6	7	8
4	**Index 1**	100	138	162	196	220	275	308	363
5	**Index 2**	45.45	62.73	73.64	89.09	100.00	125.00	140.00	165.00
6	**Profit**	3	4.14	4.86	5.88	6.6	8.25	9.24	10.89
7									
8									



24									
25									

Figure 7.3 Indices for profit at J.R. Hartman and Associates

Worked example 7.4 continued

(d) If the company made a profit of €4.86 million in year 3, you find the profit in any other year from:

$$\frac{\text{profit in year } n}{\text{index in year } n} = \frac{\text{profit in year } m}{\text{index in year } m}$$

We have to use a consistent index in this calculation, so using Index 1 and setting year 3 as year m gives:

$$\text{profit in year } n$$
$$= \text{profit in year 3} \quad \frac{\text{Index 1 in year } n}{\text{Index 1 in year 3}}$$
$$= 4.86 \quad \frac{\text{Index 1 in year } n}{162}$$

Then:

$$\text{profit in year 4} = \frac{4.86 \quad \text{Index 1 in year 4}}{162}$$
$$= \frac{4.86 \quad 196}{162}$$
$$= 5.88 \text{ or €5.88 million}$$

Here we used Index 1, but you can confirm the result using Index 2:

$$\text{profit in year 4} = \frac{4.86 \quad 89.09}{73.64} = 5.88$$

Figure 7.3 shows the profits for other years.

Review questions

7.4 When should you change the base period?

7.5 The old price index for a period is 345, while a new price index is 125. In the following period, the new price index is 132. What would the old index have been?

Indices for more than one variable

Simple indices monitor changes in a single variable, but sometimes you are concerned with changes in a combination of different variables. For instance, a car owner might want to monitor the separate costs of fuel, tax, insurance and maintenance; a company might want to monitor changes in sales of different types of products; a charity might monitor its donations to different types of causes. Indices that measure changes in a number of variables are called **aggregate indices**.

Aggregate indices

For simplicity we will look at aggregate price indices, but remember that you can use the same reasoning for any other type of index. There are two obvious ways of defining an aggregate price index:

- *The mean of the separate indices for each item* – price indices are sometimes described as the price relatives, so this average of the separate indices is called the **mean price relative index**:

$$\text{mean price relative index for period } n = \frac{\text{sum of separate indices for period } n}{\text{number of indices}}$$

- *An index based on the total cost* – this adds all prices together and calculates an index for this total price, called a **simple aggregate index**:

$$\text{simple aggregate index for period } n = \frac{\text{sum of price in period } n}{\text{sum of prices in base period}} \quad 100$$

WORKED EXAMPLE 7.5

Last year the prices of coffee, tea and hot chocolate in a café were 55 pence, 28 pence and 72 pence respectively. This year the same items cost 62 pence, 32 pence and 74 pence. What are the mean price relative index and simple aggregate index for this year based on last year?

Solution

- The mean price relative index uses the price indices for each item, which are:

coffee:	62 / 55	100 = 112.7
tea:	32 / 28	100 = 114.3
hot chocolate:	74 / 72	100 = 102.8

Taking the mean of these gives:

mean price relative index
= (112.7 + 114.3 + 102.8) / 3 = 109.9

- For the simple aggregate index we add all the prices:

sum of base prices = 55 + 28 + 72
= 155

sum of current prices = 62 + 32 + 74
= 168

Then:

simple aggregate index

$$= \frac{\text{sum of current prices}}{\text{sum of base prices}} \quad 100$$

= 168 / 155   100
= 108.4

These two indices are easy to use, but they do not really give good measures. An obvious criticism – particularly of the simple aggregate index – is that it depends on the units used for each index. An aggregate index that includes the price of, say, butter per kilogram gives a different index from one that includes the price per pound – and if we use the price of butter per tonne, this is so high that it swamps the other costs and effectively ignores them. For example, if the price of a loaf of bread rises from €1 to €1.40 and the price of a tonne of butter rises from €2,684 to €2,713, it makes no sense to calculate a simple aggregate index of (2,713 + 1.40) / (2,684 + 1)   100 = 101.09.

Another weakness of the two indices is that they do not consider the relative importance of each product. If people in the café buy more tea than hot chocolate, the index should reflect this. Imagine a service company that spent $1,000 on raw materials and $1 million on wages in the base year, and this year it spends $2,000 on raw materials and $1 million on wages. Again, it makes no sense to say that the price index for raw materials is 200 and for wages is 100, so the mean price relative index is (200 + 100) / 2 = 150.

A reasonable aggregate index must take into account two factors:

- the price paid for each unit of a product
- the number of units of each product used.