MICROECONOMICS

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MICROECONOMICS

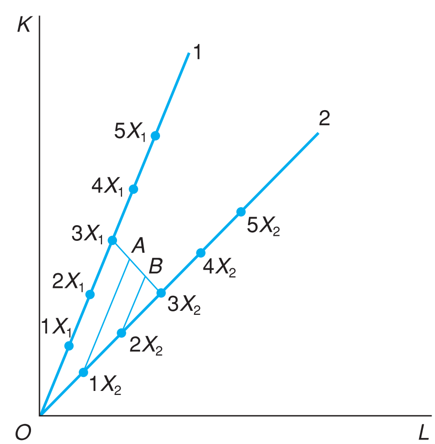
and **labour** (L). The third traditional factor of production is land, which in contemporary analysis is usually taken to include natural resources. It should be noted at the outset that just as the production of X is to be thought of as a *flow* of output, so the inputs into the production process should also be thought of in flow terms: K and L represent machine-hours and man-hours of productive services, not a stock of capital and a labour force.

The reader should not infer that it is possible to add together the services of all the manifold types of equipment and inventories used in a modern production process to give one unambiguous index number that measures the 'quantity of capital services' employed. Nor may one necessarily aggregate the hours worked by people of many different skills directly involved in production, to say nothing of those indirectly involved in a management or sales promotion role, into some unambiguously measurable quantity called 'the services of the labour force'. Nevertheless, the results of the two-input/one-output special case are both useful and often capable of being generalised and are, therefore, well worth the reader's attention.

Activities and the isoquant

We start with the most primitive concept in the analysis of production, an **activity**. Carrying on an activity means combining flows of factor services per unit of time in a particular proportion and getting a rate of flow of output from doing so. Thus, in Figure 8.1 we depict an activity that involves combining machine services and labour services in the ratio 2/1 by a line labelled 1, and another that combines them in equal proportions by a line labelled 2. It is a reasonable initial simplifying assumption that if one doubles the quantities of inputs in any particular activity one will also double output, or, to put it more generally, that equiproportional increases in inputs will lead to equiproportional increases in

Figure 8.1



Inputs K and L are combined in given proportions along rays 1 and 2 in 'activities' that yield equal increments in the output of X for equal increments to inputs. Activity 1 and activity 2 may be combined at level of output $3X$ (or at any other level of output) to make any combination of capital and labour along the line $3X_1$, $3X_2$ available as a means of producing that level of output.

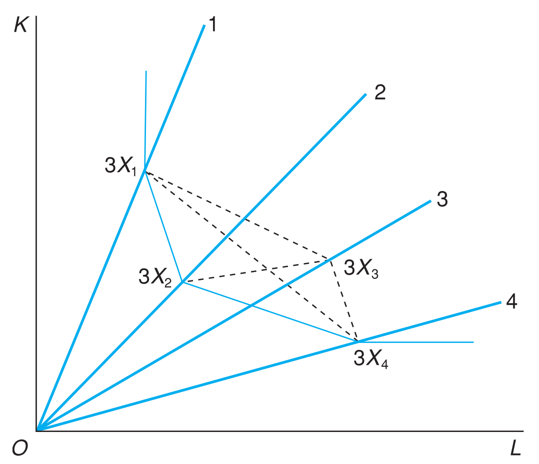
output. This is the assumption of *constant returns to scale*. An equal distance moved along each activity line in Figure 8.1 thus represents an equal increase in both inputs and output.

Now there is no obvious reason why a firm should be confined exclusively to one production activity or another. It could presumably mix them. Thus, in Figure 8.1 it could produce an output of say $3X$ units of X by using all activity 1, all activity 2, or, by combining these activities in different proportions, it could obtain that level of output by combining capital and labour services anywhere along the line joining $3X_1$ and $3X_2$. For example, $3X$ units of X could be achieved by producing $2X$ units by activity 1 and $1X$ by activity 2, thus ending up at point A. The same output could be reached by producing $1X$ unit with activity 1 and $2X$ units with activity 2, thus ending up at point B. When readers work through Chapter 13 they will recognise the similarity between this analysis and that set out there, where we consider the possibilities of mixing different brands of a particular good in order to obtain a mixture of attributes between those available from exclusive consumption of one brand or another.

There is no reason to confine the analysis to the case of a firm having just two activities available to it. In principle, the analysis can be extended to an indefinitely large number of activities, but, for the sake of clarity alone, in Figure 8.2 we depict the case of four available activities, each using capital and labour services in different proportions. Any pair of activities may be used together, and so we have linked up each available pair of activities with straight lines as we did in Figure 8.1.

At this point in the argument, even when apparently dealing with purely technological matters, we must introduce an assumption about the firm's motivation and behaviour if we are to proceed further. This assumption is that, whatever motivates those running the firm, they will never use more units of input than are necessary to get a given output. As we have drawn Figure 8.2, any output plan that utilises activity 3 requires more inputs for a given output than any plan that does not use it. This activity is said to be *technically*

Figure 8.2



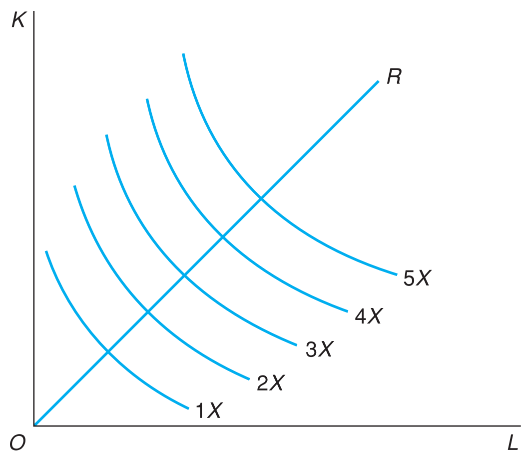
With four activities, any pair of them may be combined. Here the solid kinked line passing through $3X_1$, $3X_2$ and $3X_4$ shows the minimum combinations of capital and labour that will produce $3X$ units of output. Above activity line 1 this isoquant becomes vertical, and to the right of activity line 4 it becomes horizontal, because further additions to inputs in these directions add nothing to output.

inefficient and will not be used by a firm seeking to minimise the inputs used for any output. Moreover any combination of 1 and 4 uses more inputs than combinations of 1 and 2 and 3 and 4, or 1, 2 and 4 alone. Thus, the kinked line linking the points $3X_1$, $3X_2$ and $3X_4$ gives the locus of the minimum combinations of factor inputs required for an output of $3X$ given prevailing technology. The line becomes vertical beyond $3X_1$ and horizontal beyond $3X_4$ to indicate that further increases of capital and labour beyond these points add nothing to output.

This kinked line is a simple special case of a much used analytic device in the economics of production. The output of X is the same at any point on it, and hence it is known as an 'equal product curve' or an 'isoproduct curve' or, most frequently, an **isoquant**. The most important property of an isoquant is already implicit in the simple analysis carried out in Figure 8.2. It will never be concave towards the origin, and will in general be convex. In terms of Figure 8.2, the isoquant would have been concave to the origin if activity 3 had been used in its construction, for it would have contained the segment $3X_2$ $3X_3$ $3X_4$, but this segment is not part of the isoquant, because for any point on it there is a point on the line $3X_2$ $3X_4$ at which the same amount of output could be produced using fewer inputs.

In Figure 8.1, with two technically efficient activities we have a straight line isoquant; in Figure 8.2, with three such activities we have an isoquant that is kinked convexly to the origin. But we have already remarked that there is no need to confine ourselves to considering small numbers of activities. The more technically efficient activities there are, each using capital and labour services in different proportions, the more kinks there will be in the isoquant, and the more will it come to resemble a smooth curve, convex to the origin. Just as along a particular consumer's indifference curve we plot all those bundles of goods the consumption of which will yield equal satisfaction, so along a smooth isoquant we plot all those combinations of factor services that will yield an equal level of output. A great deal of analysis is considerably simplified, with no important loss of accuracy, if we treat the various technically efficient factor combinations that will produce a given level of output as lying along a smooth curve such as, for example, $3X$ in Figure 8.3.

Figure 8.3 **A smooth production function**



The isoquants show the maximum level of output to be had for each combination of inputs. As we move out along any ray (such as *OR*) drawn from the origin, inputs of capital and labour increase in equal proportions.

The production function

The previous section of this chapter was concerned with the various ways in which a particular level of output, 3X units of X, could be produced. The choice of this output level was, of course, quite arbitrary, and we could have carried out exactly the same analysis for any other level of output. Hence, there is not just one isoquant implicit in the foregoing analysis, but a whole family thereof, one for each conceivable level of output. A map of such isoquants is shown in Figure 8.3, which is a geometric representation of a **production function**. The output of X depends on – that is to say, is a *function* of – inputs of capital and labour services into the process. The isoquant map of Figure 8.3 tells us, for any combination of factor inputs, what the maximum attainable output of X will be. Equivalently, it also tells us, for any given level of output of X, what the minimum combinations of inputs are necessary to produce it. Provided that we are willing to think of inputs as being divisible into infinitely small units and of output as being similarly divisible, then we may also think of there being a smooth continuous isoquant passing through every point in Figure 8.3. However, as with the indifference map in consumer theory, it suffices for analytic purposes to draw only a selection of these. The rest of this chapter is devoted to looking at the properties of the production function in more detail.

Box 8.1 Deriving isoquants

We can readily derive the function for any isoquant from a Cobb–Douglas production function. Assume that the relationship between inputs (L and K) and output (X) is defined as follows:

$$X = AL^{0.75}K^{0.25} \tag{8.1.1}$$

This function is similar to that used to analyse consumer behaviour in earlier chapters. But a key difference is to include A on the right-hand side. This parameter defines 'total factor productivity' and shows the way in which output is determined, in part, by factors other than the factor inputs L and K. We can assume, without loss of generality, $A = 10$.

Using this function, any isoquant can be derived. Rearranging the production function:

$$K^{0.25} = \frac{X}{10L^{0.75}}$$

so

$$K = \frac{X^4}{10^4 L^3} \tag{8.1.2}$$

This expression defines the isoquant for any particular level of X. Fixing output to 10 units, the combinations of K and L required are defined by

$$K = \frac{1}{L^3}$$

Fixing output to 20 units, the combinations of K and L required are defined by

$$K = \frac{2^4}{L^3}$$

As an exercise, readers can assume particular levels of L, calculate the corresponding required levels of K to produce 10 or 20 units of output, and show in a graph that the values of L and K define an isoquant that has the same general shape as shown in Figure 8.3.

Long-run and short-run analysis

Before proceeding, it will be useful to distinguish in general terms between two types of analysis: **long-run** analysis, which concerns the period when all inputs can be varied simultaneously, and **short-run** analysis, when some factors are assumed to be fixed and increments to output are assumed to derive solely from changes to variable factors. If the inputs are assumed to be capital and labour, then capital is taken to be the fixed factor in the short run, and labour the variable one. If we consider Figure 8.3 as a contour map for a three-dimensional diagram, with output rising vertically from the origin O, the long-run properties of the production function describe the shape of the whole hill. Within that context, we will be concerned with **returns to scale**, which describe the slope of the hill along a fixed capital–labour ratio (such as OR) from the origin, and the degree of *substitutability* between inputs, which has to do with the curvature around the hill at a given level of output, or the shape of the isoquant. The *short-run production function* describes the relationship between output and labour input for a given level of capital, and can be imagined as being characterised by a slice through the hill at any given level of capital $\bar{K}$. The relevant property of the short-run production function concerns how output varies with labour input alone or, more generally, **returns to a factor**.

The long-run production function: some preliminaries

Algebraically, we specify the two-input production function depicted in Figure 8.3 as

$$X = f(L, K) \tag{8.1}$$

We typically assume $f(\cdot)$ to be a *monotonic* function, one in which X rises as either L or K, or both, rise. It should always be possible to produce at least as much output as in an initial situation by using at least as much of each input as in that situation. This proposition implies that the firm can always dispose costlessly of inputs which are surplus to the minimum required to produce a given level of output, a characteristic sometimes referred to as *free disposal*.

Much of the analysis which follows will draw on the relationship between two variables which characterise the production function (8.1). First, the *average product* of a factor is output per unit of time divided by factor input per unit of time: X/L for labour and X/K for capital. These ratios are sometimes referred to, rather loosely, as labour and capital *productivity* respectively. Second, the *marginal product* of a factor is the increment to output per unit of time for an increment of a factor input per unit of time, holding the other factor input constant. In the limit, when the increments become infinitesimal, they are therefore the partial derivatives of the production function (8.1). Hence $\partial X/\partial L$ is the marginal product of labour and $\partial X/\partial K$ is the marginal product of capital.

Box 8.2

Calculating factor productivities

We can use the Cobb–Douglas function defined earlier in this chapter to calculate factor productivities. With

$$X = AL^{0.75}K^{0.25} \quad (8.1.1)$$

the average products of the factors are

$$\frac{X}{L} = \frac{AL^{0.75}K^{0.25}}{L} = \frac{AK^{0.25}}{L^{0.25}} = A\left(\frac{K}{L}\right)^{0.25} \quad (8.2.1)$$

$$\frac{X}{K} = \frac{AL^{0.75}K^{0.25}}{K} = A\left(\frac{L}{K}\right)^{0.75} \quad (8.2.2)$$

The marginal products of the factors are

$$\frac{\partial X}{\partial L} = 0.75AL^{0.75-1}K^{0.25} = 0.75A\left(\frac{K}{L}\right)^{0.25} \quad (8.2.3)$$

$$\frac{\partial X}{\partial K} = 0.25AL^{0.75}K^{0.25-1} = 0.25A\left(\frac{L}{K}\right)^{0.75} \quad (8.2.4)$$

In these expressions K/L measures the capital intensity of production, and L/K the labour intensity. If capital intensity increases, we move upwards around any one isoquant indicating the increasing K/L. As this happens, the average and marginal products of labour increase and average and marginal products of capital will fall. In short, the productivity of any factor is determined (in part) by the levels of other inputs that are used. A second feature of these expressions is that as L increases $\partial X/\partial L$ declines and as K increases $\partial X/\partial K$ declines, i.e. we have diminishing marginal productivity.

Returns to scale

The concept of '**returns to scale**' refers to what happens to output when every input is increased in equal proportion. In terms of a diagram such as Figure 8.3, equiproportional increases in both inputs involve moving out along a straight line, a ray, drawn from the origin. In general, we may speak of production functions displaying decreasing returns to scale, constant returns to scale and increasing returns to scale. If successive equal increments of all factor inputs yield successively smaller increases in output, we have decreasing (or diminishing) returns to scale; if they yield equal increments in output, we have constant returns to scale and, if they yield successively increasing increments in output, we have increasing returns to scale.

The notion of returns to scale can be related to the production function (8.1) in the following way. Take equation (8.1) and note that if we start with a given level of inputs