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ADVANCED MICROECONOMIC THEORY

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Pearson

Advanced Microeconomic Theory

Consider now the price–quantity pair (p^0, q^0) on the consumer's demand curve above the competitive point in Fig. 4.6. We wish to argue that this market outcome is not Pareto efficient. To do so, we need only demonstrate that we can redistribute resources in a way that makes someone better off and no one worse off.

So, consider reducing the price of q from p^0 to p^1. What would the consumer be willing to pay for this reduction? As we now know, the answer is the absolute value of the compensating variation, which, in this case, is the sum of areas A and B in the figure. Let us then reduce the price to p^1 and take $A + B$ units of income away from the consumer. Consequently, he is just as well off as he was before, and he now demands q^1 units of the good according to his Hicksian-compensated demand.

To fulfil the additional demand for q, let us insist that the firm produce just enough additional output to meet it.

So, up to this point, we have lowered the price to p^1, increased production to q^1, and collected $A + B$ dollars from the consumer, and the consumer is just as well off as before these changes were made. Of course, the price–quantity change will have an effect on the profits earned by the firm. In particular, if $c(q)$ denotes the cost of producing q units of output, then the change in the firm's profits will be

$$
\begin{aligned}
\left[p^1q^1 - c(q^1)\right] - \left[p^0q^0 - c(q^0)\right] &= \left[p^1q^1 - p^0q^0\right] - \left[c(q^1) - c(q^0)\right] \\
&= \left[p^1q^1 - p^0q^0\right] - \int_{q^0}^{q^1} mc(q)dq \\
&= [C + D - A] - D \\
&= C - A.
\end{aligned}
$$

Consequently, if after making these changes, we give the firm A dollars out of the $A + B$ collected from the consumer, the firm will have come out strictly ahead by C dollars. We can then give the consumer the B dollars we have left over so that in the end, *both* the consumer and the firm are strictly better off as a result of the changes we have made.

Thus, beginning from the market outcome (p^0, q^0), we have been able to make both the consumer and the firm strictly better off simply by redistributing the available resources. Consequently, the original situation was not Pareto efficient.

A similar argument applies to price–quantity pairs on the consumer's Marshallian demand curve lying below the competitive point.[1] Hence, the only price–quantity pair that can possibly result in a Pareto-efficient outcome is the perfectly competitive one – and indeed it does. We shall not give the argument here because it will follow from our more general analysis in the next chapter. However, we encourage the reader to check that the particular scheme used before to obtain a Pareto improvement does not work when one begins at the competitive equilibrium. (No other scheme will produce a Pareto improvement either.)

[1]See Exercise 4.21.

Thus, our conclusion is that the only price–quantity pair yielding a Pareto-efficient outcome is the perfectly competitive one. In particular, neither the monopoly outcome nor the Cournot-oligopoly outcome is Pareto efficient.

Note well that we cannot conclude from this analysis that forcing a monopoly to behave differently than it would choose to must necessarily result in a Pareto improvement. It may well lower the price and increase the quantity supplied, but unless the consumers who are made better off by this change compensate the monopolist who is made worse off, the move will not be Pareto improving.

4.3.3 EFFICIENCY AND TOTAL SURPLUS MAXIMISATION

We have seen that consumer surplus is close to being a dollar measure of the gains going to the consumer as a result of purchasing the good in question. It is easier to find an exact way to measure the dollar value to the *producer* of selling the good to the consumer. This amount, called **producer surplus**, is simply the firm's revenue over and above its variable costs.

Now it would seem that to obtain an efficient outcome, the total surplus – the sum of consumer and producer surplus – must be maximised. Otherwise, both the producer and the consumer could be made better off by redistributing resources to increase the total surplus, and then dividing the larger surplus among them so that each obtains strictly more surplus than before.

But we must take care. Consumer surplus overstates the dollar benefits to the consumer whenever income effects are present and the good is normal. Despite this, however, under the assumption that demand is downward-sloping and the firm's marginal costs are rising, efficiency will not be achieved unless the sum of consumer and producer surplus is indeed maximised.

To see this, consider again the case of a single consumer and a single producer represented in Fig. 4.7 and consider an arbitrary price–quantity pair (p, q) on the demand curve (so that $p = p(q)$, where $p(\cdot)$ is inverse demand). Earlier we defined consumer surplus at (p, q) as the area under the demand curve and above the price p. It is easy to see that we can express that same area, and so consumer surplus, as the area under the inverse demand curve up to q minus the area of the rectangle $p(q)q$. Thus, we may express the sum of consumer and producer surplus as[2]

$$\begin{aligned} CS + PS &= \left[\int_0^q p(\xi)d\xi - p(q)q \right] + [p(q)q - tvc(q)] \\ &= \int_0^q p(\xi)d\xi - tvc(q) \\ &= \int_0^q [p(\xi) - mc(\xi)]d\xi. \end{aligned}$$

[2]The last line follows because $\int_0^q mc(\xi)d\xi = c(q) - c(0)$. Because $c(0)$ is fixed cost, and $c(q)$ is total cost, the difference $c(q) - c(0)$ is total variable cost, $tvc(q)$.

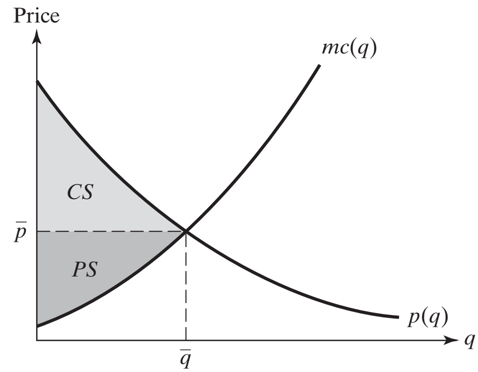

Figure 4.7. Consumer plus producer surplus is maximised at the competitive market equilibrium.

Choosing q to maximise this expression leads to the first-order condition

$$p(q) = mc(q),$$

which occurs precisely at the perfectly competitive equilibrium quantity when demand is downward-sloping and marginal costs rise, as we have depicted in Fig. 4.7.

In fact, it is this relation between price and marginal cost that is responsible for the connection between our analysis in the previous section and the present one. Whenever price and marginal cost differ, a Pareto improvement like the one employed in the previous section can be implemented. And, as we have just seen, whenever price and marginal cost differ, the total surplus can be increased.

Once again, a warning: although Pareto efficiency requires that the total surplus be maximised, a Pareto improvement need not result simply because the total surplus has increased. Unless those who gain compensate those who lose as a result of the change, the change is not Pareto improving.

We have seen that when markets are imperfectly competitive, the market equilibrium generally involves prices that exceed marginal cost. However, 'price equals marginal cost' is a necessary condition for a maximum of consumer and producer surplus. It should therefore come as no surprise that the equilibrium outcomes in most imperfectly competitive markets are *not* Pareto efficient.

EXAMPLE 4.4 Let us consider the performance of the Cournot oligopoly in Section 4.2.1. There, market demand is $p = a - bq$ for total market output q. Firms are identical, with marginal cost $c \geq 0$. When each firm produces the same output q/J, total surplus, $W \equiv cs + ps$, as a function of total output, will be

$$W(q) = \int_0^q (a - b\xi)d\xi - J\int_0^{q/J} cd\xi,$$

which reduces to

$$W(q) = aq - (b/2)q^2 - cq. \tag{E.1}$$

Because (E.1) is strictly concave, total surplus is maximised at $q^* = (a-c)/b$, where $W'(q^*) = 0$. Thus, the maximum potential surplus in this market will be

$$W(q^*) = \frac{(a-c)^2}{2b}. \tag{E.2}$$

In the Cournot-Nash equilibrium, we have seen that total market output will be $\bar{q} = J(a-c)/(J+1)b$. Clearly, $\bar{q} < q^*$, so the Cournot oligopoly produces too little output from a social point of view. Total surplus in the Cournot equilibrium will be

$$W(\bar{q}) = \frac{(a-c)^2}{2b}\frac{J^2+2J}{(J+1)^2}, \tag{E.3}$$

with a **dead weight loss** of

$$W(q^*) - W(\bar{q}) = \frac{(a-c)^2}{(J+1)^2 2b} > 0. \tag{E.4}$$

By using (E.3), it is easy to show that total surplus increases as the number of firms in the market becomes larger. Before, we noted that market price converges to marginal cost as the number of firms in the oligopoly becomes large. Consequently, total surplus rises toward its maximal level in (E.2), and the dead weight loss in (E.4) declines to zero, as $J \to \infty$. □

4.4 EXERCISES

4.1 Suppose that preferences are identical and homothetic. Show that market demand for any good must be independent of the distribution of income. Also show that the elasticity of market demand with respect to the level of market income must be equal to unity.

4.2 Suppose that preferences are homothetic but not identical. Will market demand necessarily be independent of the distribution of income?

4.3 Show that if q is a normal good for every consumer, the market demand for q will be negatively sloped with respect to its own price.

4.4 Suppose that x and y are substitutes for all but one consumer. Does it follow that the market demand for x will be increasing in the price of y?

4.5 Show that the long-run equilibrium number of firms is indeterminate when all firms in the industry share the same constant returns-to-scale technology and face the same factor prices.

4.6 A firm j in a competitive industry has total cost function $c^j(q) = aq + b_j q^2$, where $a > 0$, q is firm output, and b_j is different for each firm.

(a) If $b_j > 0$ for all firms, what governs the amount produced by each of them? Will they produce equal amounts of output? Explain.

(b) What happens if $b_j < 0$ for all firms?

4.7 Technology for producing q gives rise to the cost function $c(q) = aq + bq^2$. The market demand for q is $p = \alpha - \beta q$.

(a) If $a > 0$, if $b < 0$, and if there are J firms in the industry, what is the short-run equilibrium market price and the output of a representative firm?

(b) If $a > 0$ and $b < 0$, what is the long-run equilibrium market price and number of firms? Explain.

(c) If $a > 0$ and $b > 0$, what is the long-run equilibrium market price and number of firms? Explain.

4.8 In the Cournot oligopoly of Section 4.2.1, suppose that $J = 2$. Let each duopolist have constant average and marginal costs, as before, but suppose that $0 \leq c^1 < c^2$. Show that firm 1 will have greater profits and produce a greater share of market output than firm 2 in the Nash equilibrium.

4.9 In a **Stackelberg duopoly**, one firm is a 'leader' and one is a 'follower'. Both firms know each other's costs and market demand. The follower takes the leader's output as given and picks his own output accordingly (i.e., the follower acts like a Cournot competitor). The leader takes the follower's *reactions* as given and picks his own output accordingly. Suppose that firms 1 and 2 face market demand, $p = 100 - (q_1 + q_2)$. Firm costs are $c_1 = 10q_1$ and $c_2 = q_2^2$.

(a) Calculate market price and each firm's profit assuming that firm 1 is the leader and firm 2 the follower.

(b) Do the same assuming that firm 2 is the leader and firm 1 is the follower.

(c) Given your answers in parts (a) and (b), who would firm 1 want to be the leader in the market? Who would firm 2 want to be the leader?

(d) If each firm assumes what it wants to be the case in part (c), what are the equilibrium market price and firm profits? How does this compare with the Cournot-Nash equilibrium in this market?

4.10 (Stackelberg Warfare) In the market described in Section 4.2.1, let $J = 2$.

(a) Show that if, say, firm 1 is leader and firm 2 is follower, the leader earns higher and the follower earns lower profit than they do in the Cournot equilibrium. Conclude that each would want to be the leader.

(b) If both firms decide to act as leader and each assumes the other will be a follower, can the equilibrium be determined? What will happen in this market?

4.11 In the Cournot market of Section 4.2.1, suppose that each identical firm has cost function $c(q) = k + cq$, where $k > 0$ is fixed cost.

(a) What will be the equilibrium price, market output, and firm profits with J firms in the market?

(b) With free entry and exit, what will be the long-run equilibrium number of firms in the market?

4.12 In the Bertrand duopoly of Section 4.2.2, market demand is $Q = \alpha - \beta p$, and firms have no fixed costs and identical marginal cost. Find a Bertrand equilibrium pair of prices, (p_1, p_2), and quantities, (q_1, q_2), when the following hold.

(a) Firm 1 has fixed costs $F > 0$.

(b) Both firms have fixed costs $F > 0$.

(c) Fixed costs are zero, but firm 1 has lower marginal cost than firm 2, so $c_2 > c_1 > 0$. (For this one, assume the low-cost firm captures the entire market demand whenever the firms charge equal prices.)

4.13 Duopolists producing substitute goods q_1 and q_2 face inverse demand schedules:

$$p_1 = 20 + \tfrac{1}{2}p_2 - q_1 \qquad \text{and} \qquad p_2 = 20 + \tfrac{1}{2}p_1 - q_2,$$

respectively. Each firm has constant marginal costs of 20 and no fixed costs. Each firm is a Cournot competitor in *price*, not quantity. Compute the Cournot equilibrium in this market, giving equilibrium price and output for each good.

4.14 An industry consists of many identical firms each with cost function $c(q) = q^2 + 1$. When there are J active firms, each firm faces an identical inverse market demand $p = 10 - 15q - (J-1)\bar{q}$ whenever an identical output of $\bar{q}$ is produced by each of the other $(J-1)$ active firms.

(a) With J active firms, and no possibility of entry or exit, what is the short-run equilibrium output q^* of a representative firm when firms act as Cournot competitors in choosing output?

(b) How many firms will be active in the long run?

4.15 When firms $j = 1, \ldots, J$ are active in a monopolistically competitive market, firm j faces the following demand function:

$$q^j = (p^j)^{-2}\left(\sum_{\substack{i=1\\ i\neq j}}^{j} p_i^{-1/2}\right)^{-2}, \qquad j = 1, \ldots, J.$$

Active or not, each of the many firms $j = 1, 2, \ldots$ has identical costs,

$$c(q) = cq + k,$$

where $c > 0$ and $k > 0$. Each firm chooses its price to maximise profits, given the prices chosen by the others.

(a) Show that each firm's demand is negatively sloped, with constant own-price elasticity, and that all goods are substitutes for each other.

(b) Show that if all firms raise their prices proportionately, the demand for any given good declines.

(c) Find the long-run Nash equilibrium number of firms.

4.16 Suppose that a consumer's utility function over all goods, $u(q, \mathbf{x})$, is continuous, strictly increasing, and strictly quasiconcave, and that the price $\mathbf{p}$ of the vector of goods, $\mathbf{x}$, is fixed. Let m denote the composite commodity $\mathbf{p}\cdot\mathbf{x}$, so that m is the amount of income spent on $\mathbf{x}$. Define the utility function $\bar{u}$ over the two goods q and m as follows.

$$\bar{u}(q, m) \equiv \max_{\mathbf{x}} \; u(q, \mathbf{x}) \quad \text{s.t.} \quad \mathbf{p}\cdot\mathbf{x} \leq m.$$