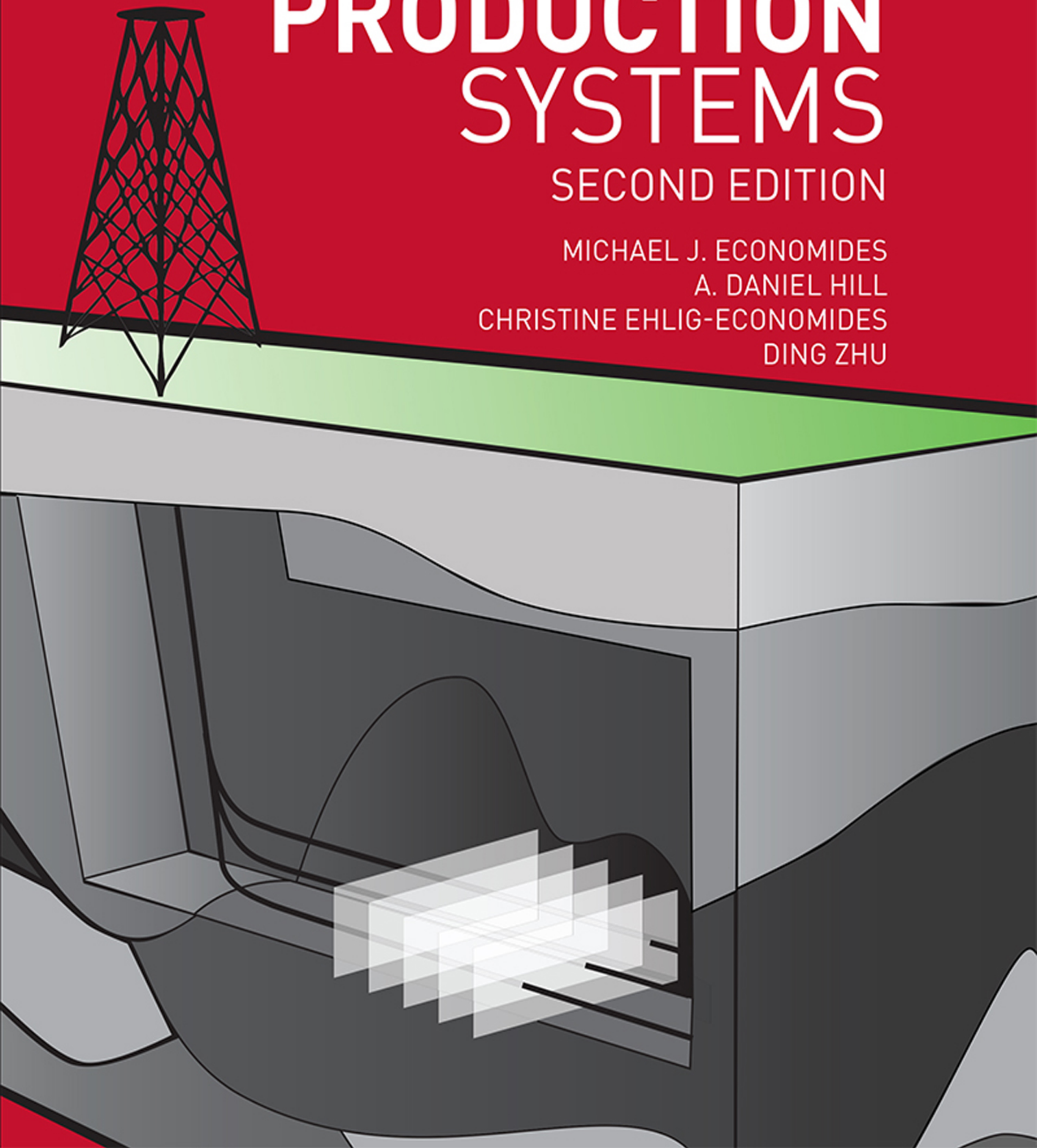
# PETROLEUM PRODUCTION SYSTEMS

## SECOND EDITION

MICHAEL J. ECONOMIDES
A. DANIEL HILL
CHRISTINE EHLIG-ECONOMIDES
DING ZHU



# PETROLEUM PRODUCTION SYSTEMS

## SECOND EDITION

find $N_{\mathrm{FR}} = 29.6$, $L_1 = 230$, $L_2 = 0.0124$, $L_3 = 0.462$, and $L_4 = 606$. Checking the flow regime limits [Equations (7-136) through (7-139)], we see that

$$0.01 \le \lambda_l < 0.4 \text{ and } L_3 < N_{\mathrm{FR}} \le L_1. \tag{7-160}$$

Then, using Equations (7-142) and (7-143),

$$C = (1 - 0.35)\ln[2.96(0.35)^{0.305}(10.28)^{-0.4473}(29.6)^{0.0978}] = 0.0351 \tag{7-161}$$

$$\psi = 1 + 0.0351\{\sin[1.8(90)] - 0.333\sin^3[1.8(90)]\} = 1.01 \tag{7-162}$$

so that, from Equation (7-140),

$$y_l = (0.454)(1.01) = 0.459. \tag{7-163}$$

Applying the Payne correction for upward flow,

$$y_l = 0.924(0.459) = 0.424. \tag{7-164}$$

The in-situ average density is

$$\overline{\rho} = y_l\rho_l + y_g\rho_g = (0.424)(49.9) + (1 - 0.424)(2.6) = 22.66\ \mathrm{lb}_m/\mathrm{ft}^3 \tag{7-165}$$

and the potential energy pressure gradient is

$$\left(\frac{dp}{dz}\right)_{\mathrm{PE}} = \frac{g}{g_c}\overline{\rho}\sin\theta = \frac{(22.66)\sin(90)}{144} = 0.157\ \mathrm{psi/ft}. \tag{7-166}$$

To calculate the frictional pressure gradient, we first compute the input fraction weighted density and viscosity from Equations (7-148) and (7-151):

$$\rho_m = (0.35)(49.9) + (0.65)(2.6) = 19.1\ \mathrm{lb}_m/\mathrm{ft}^3 \tag{7-167}$$

$$\mu_m = (0.35)(2) + (0.65)(0.0131) = 0.709\ \mathrm{cp} \tag{7-168}$$

The Reynolds number from Equation (7-150) is

$$N_{\mathrm{Re}_m} = (1488)\frac{(19.1)(13.39)(2.259/12)}{0.709} = 101{,}000. \tag{7-169}$$

For the relative roughness of 0.0006, from the Moody diagram or the Chen equation, the no-slip friction factor, $f_n$, is 0.005. Then, using Equations (7-152) through (7-154),

$$x = \frac{0.35}{(0.424)^2} = 1.95 \tag{7-170}$$

$$S = \frac{\ln(1.95)}{\{-0.0523 + 3.182\ln(1.95) - 0.8725[\ln(1.95)]^2 + 0.01853[\ln(1.95)]^4\}}$$

$$= 0.395 \tag{7-171}$$

$$f_{\text{tp}} = 0.005e^{0.395} = 0.0068 \tag{7-172}$$

From Equation (7-147), the frictional pressure gradient is

$$\left(\frac{dp}{dz}\right)_{\text{F}} = \frac{(2)(0.0068)(19.1)(13.39)^2}{(32.17)(2.259/12)} = 7.7 \text{ lb}_f/\text{ft}^3 = 0.054 \text{ psi/ft} \tag{7-173}$$

and the overall pressure gradient is

$$\left(\frac{dp}{dz}\right) = \left(\frac{dp}{dz}\right)_{\text{PE}} + \left(\frac{dp}{dz}\right)_{\text{F}} = 0.157 + 0.054 = 0.211 \text{ psi/ft.} \tag{7-174}$$

#### 7.4.3.5 The Gray Correlation

The Gray correlation was developed specifically for wet gas wells and is commonly used for gas wells producing free water and/or condensate with the gas. This correlation empirically calculates liquid holdup to compute the potential energy gradient and empirically calculates an effective pipe roughness to determine the frictional pressure gradient.

First, three dimensionless numbers are calculated:

$$N_1 = \frac{\rho_m^2 u_m^4}{g\sigma(\rho_l - \rho_g)} \tag{7-175}$$

$$N_2 = \frac{gD^2(\rho_l - \rho_g)}{\sigma} \tag{7-176}$$

$$N_3 = 0.0814\left[1 - 0.0554\ln\left(1 + \frac{730R_v}{R_v + 1}\right)\right] \tag{7-177}$$

where

$$R_v = \frac{u_{sl}}{u_{sg}}. \tag{7-178}$$

The liquid holdup correlation is

$$y_l = 1 - (1 - \lambda_l)(1 - \exp(f_l)) \tag{7-179}$$

where

$$f_l = -2.314\left[N_1\left(1 + \frac{205}{N_2}\right)\right]^{N_3}. \tag{7-180}$$

The potential energy pressure gradient is then calculated using the in-situ average density.

To calculate the frictional pressure gradient, the Gray correlation uses an effective pipe roughness to account for liquid along the pipe walls. The effective roughness correlation is

$$k_e = k_o \text{ for } R_v > 0.007 \tag{7-181}$$

or

$$k_e = k + R_v\left\{\frac{k_o - k}{0.007}\right\} \text{ for } R_v < 0.007 \tag{7-182}$$

where

$$k_o = \frac{12.92738\sigma}{\rho_m u_m^2} \tag{7-183}$$

and $k$ is the absolute roughness of the pipe. The constant in Equation 7-183 is for all variables in consistent units. For oilfield units of dynes/cm for $\sigma$, $lb_m/ft^3$ for $\rho$, and ft/sec for $u_m$,

$$k_o = \frac{0.285\sigma}{\rho_m u_m^2}. \tag{7-184}$$

The effective roughness is used to calculate the relative roughness by dividing by the pipe diameter. The friction factor is obtained using this relative roughness and a Reynolds number of $10^7$.

**Example 7-10 Pressure Gradient Calculation Using the Gray Method**

An Appendix C gas well is producing 2 MMSCF/day of gas with 50 bbl of water produced per MMSCF of gas. The surface tubing pressure is 200 psia and the temperature is 100°F. The gas–water surface tension is 60 dynes/cm, and the pipe relative roughness is 0.0006. Calculate the pressure gradient at the top of the tubing, neglecting any kinetic energy contribution to the pressure gradient, At this location, the water density is 65 $\text{lb}_\text{m}/\text{ft}^3$ and the viscosity is 0.6 cp.

***Solution***

First, we need to determine the $Z$ factor for this gas. $p_{pr} = 0.298$ ($p/p_{pc} = 200/671$) and $T_{pr} = 1.49$ ($T/T_{pc} = 560/375$). From Figure 4-1, $Z = 0.97$. The flowing area $A = 0.0278\ \text{ft}^2$ if 2 7/8-in. tubing is used.

The superficial velocities are calculated as

$$u_{sl} = \frac{q_l}{A} = \left[\frac{(100\ \text{STB/day})(5.615\ \text{ft}^3/\text{bbl})(1\ \text{d}/86{,}400\ \text{s})}{0.0278\ \text{ft}^2}\right] = 0.2335\ \text{ft/s}. \tag{7-185}$$

The gas superficial velocity is calculated from the volumetric flow rate at standard conditions with Equation (7-45),

$$u_{sg} = \frac{4}{\pi(2.259/12)^2}(2 \times 10^6\ \text{ft}^3/\text{d})(0.97)\left(\frac{460 + 100}{460 + 60}\right)\left(\frac{14.7}{200}\right)\frac{\text{d}}{86{,}400\ \text{s}} = 63.9\ \text{ft/s}. \tag{7-186}$$

The mixture velocity is

$$u_m = u_{sl} + u_{sg} = (0.2335 + 63.9) = 64.09\ \text{ft/s}. \tag{7-187}$$

And the input fraction of liquid is

$$\lambda_l = \frac{u_{sl}}{u_m} = \frac{0.2335}{63.9} = 0.0036. \tag{7-188}$$

The gas density is calculated from Equation (7-44),

$$\rho_g = \frac{(28.97)(0.65)(200\ \text{psi})}{(0.97)(10.73\ \text{psi-ft}^3/\text{lb-mol-°R})(560\text{°R})} = 0.65\ \text{lb}_m/\text{ft}^3. \tag{7-189}$$

The input fraction weighted density is calculated from Equation (7-148),

$$\rho_m = (0.0036)(65) + (1 - 0.0036)(0.65) = 0.88\ \text{lb}_m/\text{ft}^3. \tag{7-190}$$

From Equations (7-175) and (7-176), we calculate $N_1$ and $N_2$,

$$N_1 = \frac{\rho_m^2 u_m^4}{g\sigma(\rho_l - \rho_g)}$$

$$= \left[\frac{(0.88\ \text{lb}_\text{m}/\text{ft}^3)^2(64.09\ \text{ft/s})^4}{(32.17\ \text{ft-lb}_\text{m}/\text{lb}_\text{f}\text{-s}^2)(60\ \text{dynes/cm})\left(6.85\times10^{-5}\dfrac{\text{lb}_\text{f}/\text{ft}}{\text{dynes/cm}}\right)(65\ \text{lb}_\text{m}/\text{ft}^3 - 0.65\ \text{lb}_\text{m}/\text{ft}^3)}\right]$$

$$= 1.537 \times 10^6 \quad \textbf{(7-191)}$$

$$N_2 = \frac{gD^2(\rho_l - \rho_g)}{\sigma} = \left[\frac{(32.17\text{ft-lb}_\text{m}/\text{lb}_\text{f}\text{-s}^2)(2.259/12\ \text{ft})^2(65\ \text{lb}_\text{m}/\text{ft}^3 - 0.65\ \text{lb}_\text{m}/\text{ft}^3)}{(60\ \text{dynes/cm})\left(6.85\times10^{-5}\dfrac{\text{lb}_\text{f}/\text{ft}}{\text{dynes/cm}}\right)}\right]$$

$$= 1.785 \times 10^4 \quad \textbf{(7-192)}$$

We calculate $R_v$ and $N_3$ with Equations (7-178) and (7-177),

$$R_v = \frac{u_{sl}}{u_{sg}} = \frac{0.2335}{63.9} = 0.0037 \quad \textbf{(7-193)}$$

$$N_3 = 0.0814\left[1 - 0.0554\ \ln\left(1 + \frac{730 \times 0.0037}{0.0037 + 1}\right)\right] = 0.0755 \quad \textbf{(7-194)}$$

With Equations (7-180) and (7-179), $f_1$ and $y_1$ are

$$f_l = -2.314\left[N_1\left(1 + \frac{205}{N_2}\right)\right]^{N_3} = -6.7942 \quad \textbf{(7-195)}$$

$$y_l = 1 - (1 - \lambda_l)(1 - \exp(f_l)) = 0.0048 \quad \textbf{(7-196)}$$

The in-situ average density is

$$\overline{\rho} = y_l\rho_l + (1 - y_l)\rho_g = (0.0048)(65) + (1 - 0.0048)(0.65) = 0.9524\ \text{lb}_m/\text{ft}^3. \quad \textbf{(7-197)}$$

The potential energy pressure gradient is

$$\left(\frac{dp}{dz}\right)_{\text{PE}} = \frac{g}{g_c}\overline{\rho}\sin\theta = \frac{(0.9524)\sin(90°)}{144} = 0.0066\ \text{psi/ft}. \quad \textbf{(7-198)}$$

The absolute roughness of the pipe is $k = \varepsilon D = (0.0006)(2.259)/12 \text{ ft} = 0.000113 \text{ ft}$.

Using Equation (7-184),

$$k_o = \frac{0.285(60 \text{ dynes/cm})}{(0.88 \text{ lb}_m/\text{ft}^3)(64.09 \text{ ft/s})^2} = 0.0047 \text{ ft}. \tag{7-199}$$

For $R_v = 0.0037 < 0.007$,

$$k_e = k + R_v\left\{\frac{k_o - k}{0.007}\right\} = 0.0025 \text{ ft}. \tag{7-200}$$

The effective relative roughness is $k_e/D = 0.0134$. With Equation (7-37), the Fanning friction factor is 0.0105.

From Equation (7-147), the frictional pressure gradient is

$$\left(\frac{dp}{dz}\right)_{\text{F}} = \frac{(2)(0.0105)(0.88)(64.09)^2}{(32.17)(2.259/12)(144)} = 0.087 \text{ psi/ft} \tag{7-201}$$

and the overall pressure gradient is

$$\left(\frac{dp}{dz}\right) = \left(\frac{dp}{dz}\right)_{\text{PE}} + \left(\frac{dp}{dz}\right)_{\text{F}} = 0.0066 + 0.087 = 0.0936 \text{ psi/ft}. \tag{7-202}$$

### 7.4.4 Pressure Traverse Calculations

We have examined several methods for calculating the pressure gradient, *dp/dz,* which can be applied at any location in a well. However, our objective is often to calculate the overall pressure drop, $\Delta p$, over a considerable distance, and over this distance the pressure gradient in gas–liquid flow may vary significantly as the downhole flow properties change with temperature and pressure. For example, in a well such as that pictured in Figure 7-15, in the lower part of the tubing the pressure is above the bubble point and the flow is single-phase oil. At some point, the pressure drops below the bubble point and gas comes out of solution, causing gas–liquid bubble flow; as the pressure continues to drop, other flow regimes may occur farther up the tubing.

Thus, we must divide the total distance into increments small enough that the flow properties, and hence the pressure gradient, are almost constant in each increment. Summing the pressure drop in each increment, we obtain the overall pressure drop. This stepwise calculation procedure is generally referred to as a pressure traverse calculation.