Introduction to Wireless Digital Communication

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The connection between baseband and passband signals is important in viewing wireless communication systems through the lens of signal processing. Throughout this book, when dealing with wireless communication signals, only their complex baseband equivalents are considered. Issues related to imperfections in the upconversion and downconversion, due to differences between the carrier frequency at the transmitter and the receiver, are explored further in Chapter 5.

3.3.3 Complex Baseband Equivalent Channel

Thus far we have established a convenient representation for generating passband signals from complex baseband signals, and for extracting complex baseband signals from passband signals. Upconversion is used at the transmitter to create a passband signal, and downconversion is used at the receiver to extract a baseband signal from a passband signal. These are key operations performed by the analog front end in a wireless communication system.

From a signal processing perspective, it is convenient to work exclusively with complex baseband equivalent signals. A good model for the wireless channel is an LTI system as argued in Section 3.1.2. This system, though, is applied to the transmitted passband signal $x_{\mathrm{p}}(t)$ to produce the received passband signal

$$y_{\mathrm{p}}(t) = \int h_{\mathrm{c}}(t-\tau)x_{\mathrm{p}}(\tau)\mathrm{d}\tau \tag{3.330}$$

as illustrated in Figure 3.18. Because it is at passband, the input-output relationship in (3.330) is a function of the carrier frequency. Based on the results in Section 3.3.2, it would be convenient to have an equivalent baseband representation that is just a function of the complex envelopes of the input signal and the output signal for some baseband channel $h(t)$. In other words, it would be nice to find a relationship

$$y(t) = \int h(t-\tau)x(\tau)\mathrm{d}\tau \tag{3.331}$$

for some suitable $h(t)$. The complex baseband equivalent channel $h(t)$ thus acts on the input signal $x(t)$ to produce $y(t)$, all at baseband.

In this section, we summarize the steps in the derivation of the complex baseband equivalent channel $h(t)$ from the real channel $h_{\mathrm{c}}(t)$ that is not bandlimited. The idea illustrated in Figure 3.18 is to recognize that only a portion of the channel response impacts the passband signal. Then an equivalent passband channel response can be considered as illustrated in Figure 3.19, without any loss. Finally, the passband channel is converted to its baseband equivalent in Figure 3.20.

Now we provide the mathematical steps for deriving the complex baseband equivalent channel. We work with the frequency domain, following Figure 3.20. The objective is to find the baseband equivalent channel $\mathsf{h}(f)$ such that $\mathsf{y}(f) = \mathsf{h}(f)\,\mathsf{x}(f)$. Attention is needed to the scaling factors in the derivation.

Consider the received signal

$$\mathsf{y}_{\mathrm{p}}(f) = \mathsf{h}_{\mathrm{c}}(f)\mathsf{x}_{\mathrm{p}}(f). \tag{3.332}$$

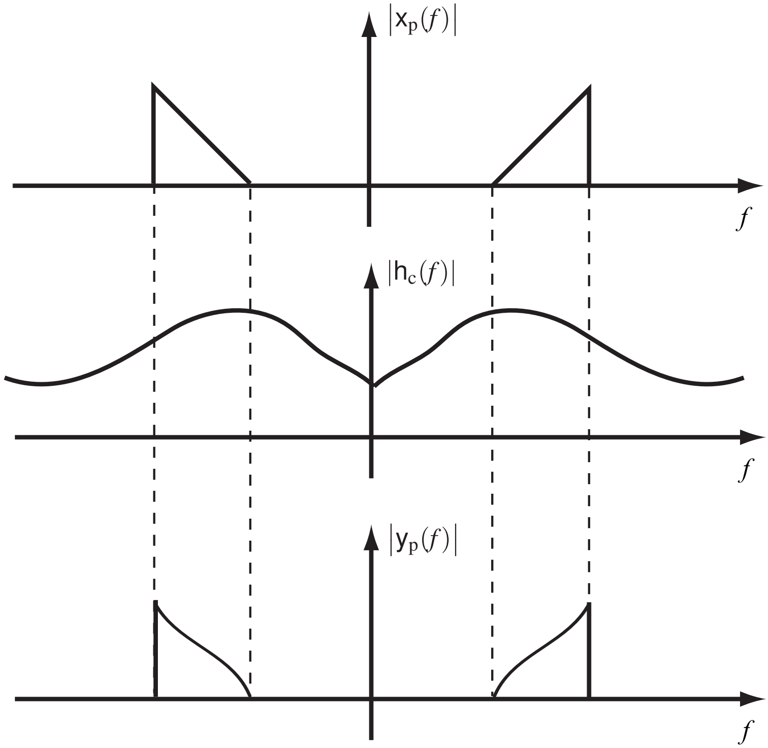

Figure 3.18 Convolution between the passband signal and the channel. In this case the total response of the channel $h_c(f)$ is illustrated.

Because $x_p(f)$ is a passband signal, only the part of $h_c(f)$ that is within the passband of $x_p(f)$ is important. Let us denote this portion of the channel as $h_p(f)$.

The passband channel $h_p(f)$ is derived by filtering $h(f)$ with an ideal passband filter. Assuming that $x_p(f)$ has a passband bandwidth of B, the corresponding ideal passband filter with bandwidth of B centered around f_c is

$$p(f) = \text{rect}\left(\frac{f - f_c}{B}\right) + \text{rect}\left(\frac{-(f + f_c)}{B}\right) \tag{3.333}$$

$$= \text{rect}\left(\frac{f - f_c}{B}\right) + \text{rect}\left(\frac{f + f_c}{B}\right) \tag{3.334}$$

or equivalently in the time domain is

$$p(t) = 2B\cos(2\pi f_c t)\text{sinc}(Bt). \tag{3.335}$$

Since $x_p(f) = x_p(f)p(f)$, it follows that

$$y_p(f) = h_c(f)p(f)x_p(f) \tag{3.336}$$

$$= h_p(f)x_p(f) \tag{3.337}$$

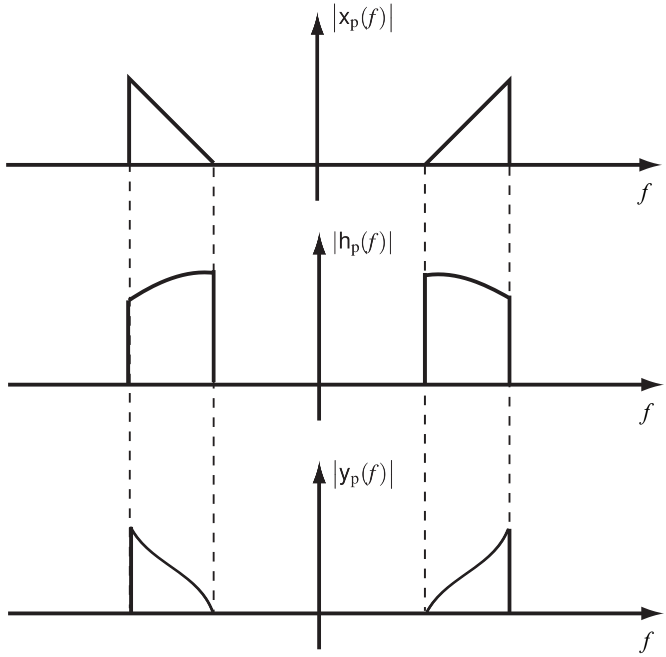

Figure 3.19 Convolution between the passband signal and the channel

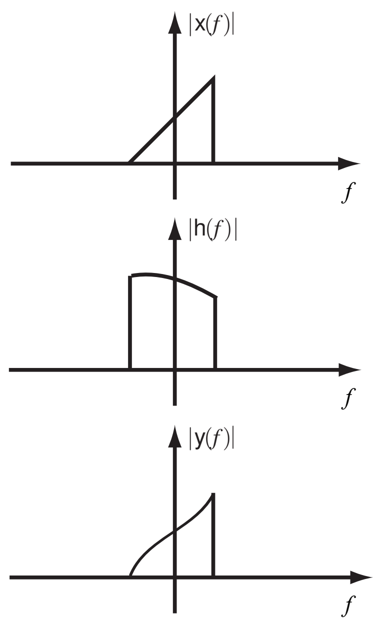

Figure 3.20 Convolution between the baseband signal and the baseband equivalent channel

where

$$\mathsf{h}_{\mathrm{p}}(f) = \mathsf{p}(f)\mathsf{h}_{\mathrm{c}}(f) \tag{3.338}$$

is the passband filtered channel.

Now suppose that $\mathsf{h}(f)$ is the complex baseband equivalent channel so that $\mathsf{y}(f) = \mathsf{h}(f)\mathsf{x}(f)$. The passband signal obtained using (3.318) is then

$$\mathsf{y}_{\mathrm{p}}(f) = \frac{1}{2}\mathsf{h}(f - f_{\mathrm{c}})\mathsf{x}(f - f_{\mathrm{c}}) \;+\; \frac{1}{2}\mathsf{h}^*(-f - f_{\mathrm{c}})\mathsf{x}^*(-f - f_{\mathrm{c}}). \tag{3.339}$$

But it is also true that

$$\mathsf{y}_{\mathrm{p}}(f) = \mathsf{h}_{\mathrm{p}}(f)\mathsf{x}_{\mathrm{p}}(f). \tag{3.340}$$

Let $\mathsf{h}_{\mathrm{b}}(f)$ be the complex baseband equivalent of $\mathsf{h}_{\mathrm{p}}(f)$. Substituting for $\mathsf{h}_{\mathrm{p}}(f)$ and $\mathsf{x}_{\mathrm{p}}(f)$ again using (3.318),

$$\mathsf{y}_{\mathrm{p}}(f) = \left(\frac{1}{2}\mathsf{h}_{\mathrm{b}}(f - f_{\mathrm{c}}) + \frac{1}{2}\mathsf{h}_{\mathrm{b}}{}^*(-f - f_{\mathrm{c}})\right)\left(\frac{1}{2}\mathsf{x}(f - f_{\mathrm{c}}) + \frac{1}{2}\mathsf{x}^*(-f - f_{\mathrm{c}})\right) \tag{3.341}$$

$$= \frac{1}{4}\mathsf{h}_{\mathrm{b}}(f - f_{\mathrm{c}})\mathsf{x}(f - f_{\mathrm{c}}) + \frac{1}{4}\mathsf{h}_{\mathrm{b}}{}^*(-f - f_{\mathrm{c}})\mathsf{x}^*(-f - f_{\mathrm{c}}). \tag{3.342}$$

Equating terms in (3.342) and (3.339), it follows that

$$\mathsf{h}(f) = \frac{1}{2}\mathsf{h}_{\mathrm{b}}(f). \tag{3.343}$$

The factor of 1/2 arises because the passband channel was obtained by passband filtering a non-bandlimited signal, then downconverting. With similar calculations as in (3.329),

$$\mathsf{h}(f) = \frac{1}{2}\;2\,\mathrm{rect}\left(\frac{f}{B}\right)\mathsf{h}_{\mathrm{p}}(f + f_{\mathrm{c}}) \tag{3.344}$$

$$= \mathrm{rect}\left(\frac{f}{B}\right)\left[\mathrm{rect}\left(\frac{f}{B}\right)\mathsf{h}_{\mathrm{c}}(f + f_{\mathrm{c}}) \;+\; \mathrm{rect}\left(\frac{f + f_{\mathrm{c}} + f_{\mathrm{c}}}{B}\right)\mathsf{h}_{\mathrm{c}}(f - f_{\mathrm{c}})\right] \tag{3.345}$$

$$= \mathrm{rect}\left(\frac{f}{B}\right)\mathsf{h}_{\mathrm{c}}(f + f_{\mathrm{c}}). \tag{3.346}$$

Compared with (3.329), the factor of 1/2 cancels the factor of 2 in (3.346). The entire process of generating the complex baseband equivalent channel is illustrated in Figure 3.21.

The time-domain response follows directly from the Fourier transform properties in Table 3.1:

$$h(t) = B\mathrm{sinc}(Bt) * h_{\mathrm{c}}(t)e^{\mathrm{j}2\pi f_{\mathrm{c}} t} \tag{3.347}$$

$$= B\int \mathrm{sinc}(B(t-\tau))h_{\mathrm{c}}(\tau)e^{\mathrm{j}2\pi f_{\mathrm{c}}\tau}\mathrm{d}\tau. \tag{3.348}$$

Essentially, the complex baseband equivalent channel is a demodulated and filtered version of the continuous-time channel $h_{\mathrm{c}}(t)$. The time-domain operations are illustrated in Figure 3.22. Several example calculations are provided in Examples 3.37 and 3.38.

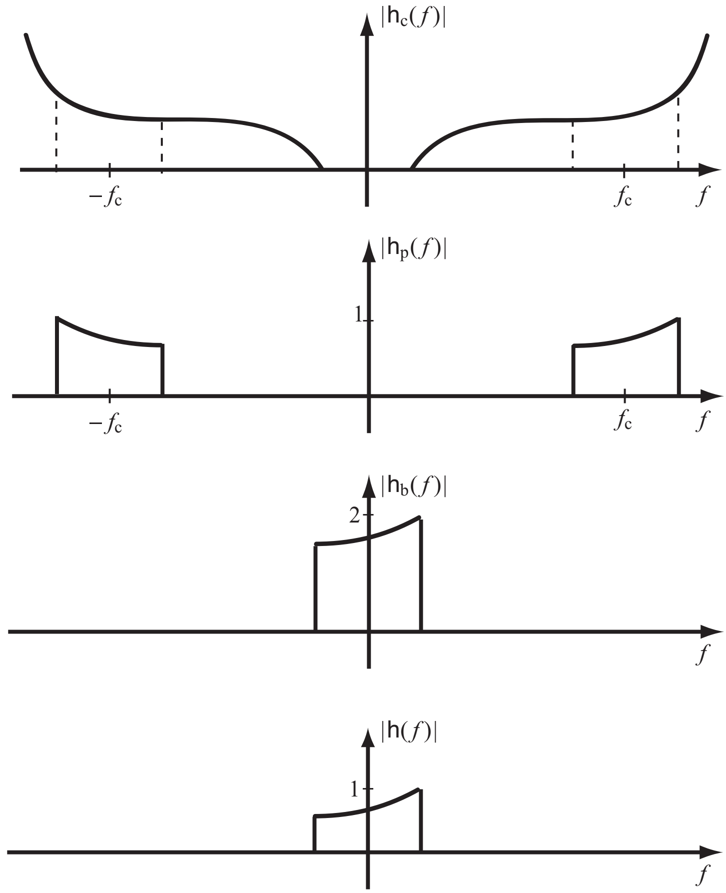

Figure 3.21 Frequency-domain illustration of the transformation from $h_c(f)$ to $h_p(f)$ to $h_b(f)$ to $h(f)$. An arbitrary scaling is selected in $h_p(f)$ just to illustrate subsequent rescalings.

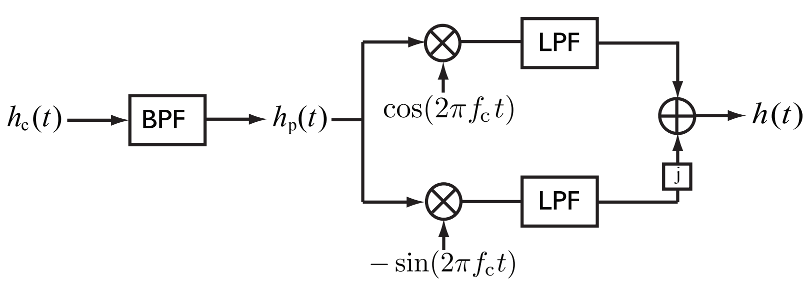

Figure 3.22 Creating the complex baseband equivalent by demodulating the channel

Example 3.37 Suppose that the signal has to propagate over a distance of 100m to arrive at the receiver and undergoes an attenuation of 0.1. Find a linear time-invariant model for this channel $h_{\rm c}(t)$, then find the passband channel $h_{\rm p}(t)$ and the baseband equivalent channel $h(t)$. The baseband signal bandwidth is 5MHz and the carrier frequency is $f_{\rm c} = 2$GHz.

Answer: Consider the speed of propagation to be $c = 3 \times 10^8 m/s$. Then the delay between the transmitter and the receiver is $\tau_{\rm d} = 100/c = 1/3\mu{\rm s} = 1/3 \times 10^{-6}{\rm s}$. Consequently, the channel can be modeled as

$$h_{\rm c}(t) = 0.1\delta(t - 1/3 \times 10^{-6}). \tag{3.349}$$

The baseband signal bandwidth is 5MHz; thus the passband bandwidth is $B = 10$MHz. Now

$$\begin{aligned} h_{\rm p}(t) &= h_{\rm c}(t) * p(t) &(3.350)\\ &= \int p(t-\tau)h_{\rm c}(\tau){\rm d}\tau &(3.351)\\ &= 0.1 \int p(t-\tau)\delta(t - 1/3 \times 10^{-6}){\rm d}\tau &(3.352)\\ &= 0.1\ p(t - 1/3 \times 10^{-6}) &(3.353)\\ &= 2 \times 0.1 \times 10^7 \cos(2\pi 2 \times 10^9(t - 1/3 \times 10^{-6})){\rm sinc}(10^7(t - 1/3 \times 10^{-6})). &(3.354) \end{aligned}$$

Similarly, to find the baseband channel, we use (3.348):

$$\begin{aligned} h(t) &= 0.1 \times 10^7 \int {\rm sinc}(10^7(t-\tau))\delta(\tau - 1/3 \times 10^{-6})e^{-{\rm j}2\pi \times 2 \times 10^9 \tau}{\rm d}\tau &(3.355)\\ &= 0.1 \times 10^7 {\rm sinc}(10^7(t - 1/3 \times 10^{-6}))e^{-{\rm j}2\pi\frac{2}{3}\times 10^3}. &(3.356) \end{aligned}$$

Simplifying further given the numbers in the problem,

$$h(t) = 10^6 {\rm sinc}(10^7(t - 1/3 \times 10^{-6}))e^{-{\rm j}\pi\frac{4}{3}}. \tag{3.357}$$

Example 3.38 Consider a wireless communication system with the carrier frequency of $f_{\rm c} = 900$MHz and the absolute bandwidth of $B = 5$MHz. The two-path channel model is

$$h_{\rm c}(t) = \delta(t - 2 \times 10^{-6}) + 0.5\delta(t - 5 \times 10^{-6}). \tag{3.358}$$

- Determine the Fourier transform of $h_{\rm c}(t)$.

  Answer: Let $t_1 = 2 \times 10^{-6}$ and $t_2 = 5 \times 10^{-6}$ and rewrite the channel response in the time domain as

$$h_{\rm c}(t) = \delta(t - t_1) + 0.5\delta(t - t_2). \tag{3.359}$$