CSCMP
COUNCIL OF SUPPLY CHAIN MANAGEMENT PROFESSIONALS

THE DEFINITIVE GUIDE TO INVENTORY MANAGEMENT

Principles and Strategies for the Efficient Flow of Inventory across the Supply Chain

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Table 3-6 Probability Mass

Demand	Cumulative Poisson	Probability
0	0.60653066	0.60653066
1	0.90979599	0.30326533
2	0.98561232	0.07581633
3	0.99824838	0.01263606
4	0.99982788	0.00157951
5	0.99998584	0.00015795

So, if we set ROP to 2, we will have a PPIS of 0.986. The probability of selling 2 during the lead time is found in the last column, 0.076.

In reality, many hybrid replenishment processes exist. Let's first consider a hybrid to the (T,OUL) replenishment process. Suppose we took an existing (T,OUL) process and added a reorder point to it. So, the idea would be that when we got to a review point, we would only order the difference between the inventory position and the OUL if the inventory position were below the ROP. Would the new (T,OUL,ROP) process have more, less, or the same average inventory level? Without the ROP, we would order at every review time, but with the ROP we might not order at some of the review times. Consequently, we would expect less inventory on average if a ROP were added to an existing (T,OUL) process. There are many other hybrid processes as well. Many of them are difficult to model, so it is good to understand these two example replenishment processes (T,OUL) and (Q,ROP) and to think about how variations affect inventory levels and protection periods in particular. When a new process is created, it is always good to think carefully about how new steps affect risk of failure and the protection period in general.

As we have already mentioned, being able to represent your ideas in an inventory graph can facilitate careful and rigorous thinking about how the process will perform and where it will have potential failures. When you draw these graphs you can use them to brainstorm various scenarios that might occur. This can help you think through how various steps affect the protection period. It can be more helpful if you have a colleague who is also competent in inventory management and who can also use inventory graphs to discuss new inventory systems or changes to existing systems. In addition to drawing inventory graphs, it is also useful to have someone who can build a discrete event simulation model of the process.

Discrete event simulation can be done in Excel or a discrete event simulation software package. It allows you to model business processes such as replenishment processes and allows for uncertainty to be brought into things like demand and lead time. After a lot of discussion with the inventory graphs, carefully designed flow charts can be created. These are used to then create the discrete event simulation. In the process of developing the discrete event simulation of the inventory process, questions will arise about details of how the process works. The actual development of the discrete event simulation requires a great deal of specificity. Usually the first few times these models are run, problems occur because something has not been specified. This leads to additional refinement of thinking about the process that is being created or modified. In the previous discussion about (T,OUL) and (Q,ROP) processes, we gave specific formulas for estimating OUL or ROP. Even for these processes, for many distributions, we cannot write a specific formula for OUL or ROP. In those cases, discrete event simulation is needed. It gets even more complicated for many hybrid processes, and then discrete event simulation is about the only alternative. There are many different levels of skill in discrete event simulation. However, it is useful for someone involved in inventory management to at least be able to make rough cut discrete event simulation models in Excel.

Expected Units Out Per Replenishment Cycle

We now return to the (Q,ROP) process and consider the optimal order quantity, but first we must consider the total cost of a given order quantity. For a given order quantity, the expected cost of carrying cycle stock is (Q/2)hc.[9] In addition, the expected cost of carrying safety stock is (ROP – EDDLT)hc. The number of orders placed per year is (D/Q), so if the cost of each order is S, then the annual ordering cost is (D/Q)S. During each lead time there is a chance of stocking out and losing sales. The expected number of units out of stock per replenishment cycle is

$$\mathrm{U(ROP)} = \int_{x=ROP}^{\infty} (x - ROP) f(x) dx$$

This is called the loss integral.[10] We show you how to calculate this with a normal distribution in Excel.

$$\mathrm{U(ROP)} = \int_{x=ROP}^{\infty} (x - ROP) f(x) dx$$

$$= \sigma_{DDLT} NORMDIST(Z,0,1,0) - (ROP - EDDLT)(1 - NORMDIST(Z,0,1,1))$$

In this formula, Z is the number of standard deviations above the mean demand during lead time that is represented by the ROP. If you use =NORMDIST(ROP,EDDLT, σ_{DDLT},1) to find the PPIS, you can then use =NORMSINV(PPIS) to find Z. Then you can apply the loss integral formula.

Using the example from Table 3-1, suppose we set ROP to 52 units.

$$=\text{NORMDIST(ROP,EDDLT, } \sigma_{DDLT} \text{,1)}$$

=NORMDIST(52,49,12,1) returns a value of 0.5987. Then using =NORMSINV(PPIS) =NORMSINV(0.5987) we get 0.25, which is the value of Z.

Now, to get the expected number of units out of stock per replenishment cycle, we have

$$\text{U(ROP)} = \int_{x=ROP}^{\infty} (x - ROP) f(x) dx$$

$$= \sigma_{DDLT} NORMDIST(Z,0,1,0) - (ROP - EDDLT)(1 - NORMDIST(Z,0,1,1))$$

$$= 12 * NORMDIST(0.25,0,1,0) - (52 - 49)(1 - NORMDIST(0.25,0,1,1))$$

This returns a value of 3.4 units per replenishment cycle. Now, suppose the expected cost of a unit out of stock is $10 per unit out of stock. Then every time we place an order our expected cost of lost sales is about $34. This is an additional ordering-related cost. Suppose you only use truckload[11] for transportation and that each truckload costs $150. Other ordering related costs are $20, including accounts payable variable costs, receiving, and so on. Then the total cost associated with placing an order is $150 + $34 + $20 = $204 per order.

Total Annual Cost as a Function of Order Quantity

Since there are (D/Q) replenishments per year,[12] the expected number of units out of stock per year is (D/Q)U(ROP). Suppose the cost per unit is m, then the expected cost of out of units out of stock per year is m(D/Q)U(ROP). The in-transit holding cost is (L/365)*D*hc. If lead time is in days, then 365 days is used for the denominator; if lead time is in weeks, then 52 weeks is used for the denominator, and so on. So the expected cost is

$$C(Q) = Dc + \left(\frac{D}{Q}\right)(S + mU(ROP)) + \left(\left(\frac{Q}{2}\right) + (ROP - EDDLT) + \left(\frac{LD}{365}\right)\right)hc$$

We now discuss where transportation costs fit into this analysis. Truckload (TL) costs are based on point-to-point service. That is, the amount charged for the transportation is based on the rate a carrier changes from point A to point B. If TL is used, the same cost is incurred regardless of how much is shipped as long as it is less than the TL capacity. In this case, the cost of a TL is added to the ordering cost because each time an order is placed, the transportation cost of a TL must be paid. On the other hand, if less than truckload (LTL) is used, then the cost is based on the weight. LTL rates[13] are first based upon product class from National Motor Freight Classification (NMFC) published by the National Motor Freight Traffic Association (NMFTA). Then the carrier's tariff is used based on the origin and destination. Next the rates are discounted by the carrier. Finally the cost is based on the weight shipped. Consequently, transportation is based on weight shipped, which can be translated into a cost per unit. As a result, the transportation cost is added to the value of the item, c. So, if TL is used, transportation costs are a part of the ordering cost, whereas if LTL is used, transportation costs are a part of the unit cost. In general, if the transportation cost is based on the order not on the quantity, the transportation cost goes in the ordering cost, whereas if it is based on the amount shipped, it goes in the unit cost.

Taking the derivative of the total cost function above with respect to Q, and setting it equal to zero, we have

$$\frac{\partial C}{\partial Q} = \frac{hc}{2} - \left(\frac{D}{Q^2}\right)\left(S + mU\left(ROP\right)\right) = 0$$

Solving for Q, we have the EOQ.[14]

$$Q = \sqrt{\frac{2D\left(S + mU\left(ROP\right)\right)}{hc}}$$

Taking the second derivative we find

$$\frac{\partial^2 C}{\partial Q^2} = 2\left(\frac{D}{Q^3}\right)\left(S + mU\left(ROP\right)\right) > 0$$

Meaning that the function is convex, so we have found a global, unique minimum.

All of this can be done with discrete empirical distributions as well.

The optimal solution may result in using TL but not filling the truck all the way. That is, it might be less expensive to use TL than LTL, and, at the same time, it is not optimal to utilize the transportation to 100 percent. Usually, however, more than one item is shipped on the truck.

When evaluating ordering decisions and transportation decisions, there is a trade-off between inventory holding costs and transportation costs that must be taken into account.

Let

$$I = \left(\frac{Q}{2}\right) + (ROP - EDDLT) + \left(\frac{LD}{365}\right)$$

and

$$B = S + mU(ROP) \text{ then}$$

$$C(Q) = Dc + \left(\frac{D}{Q}\right)B + Ihc$$

So B is the fixed cost of ordering, and I is the amount of inventory.

Figure 3-13 illustrates the cost trade-offs associated with various levels of Q in a (Q,ROP) continuous review replenishment process.

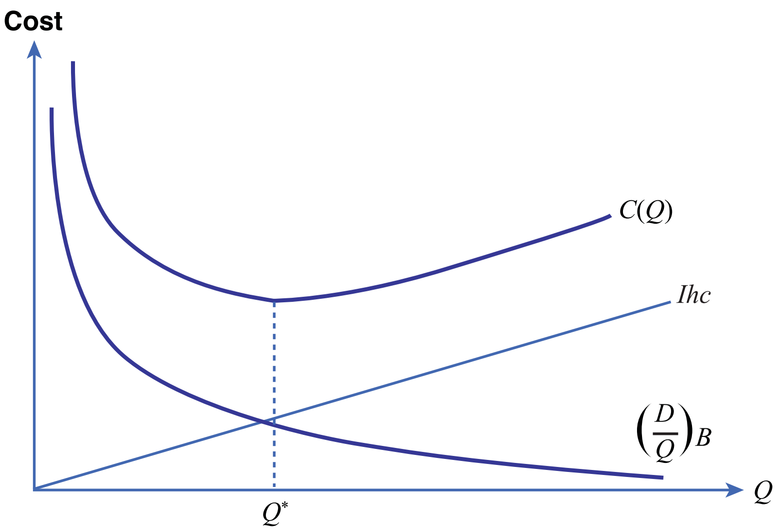

Figure 3-13 Cost tradeoffs

Generally, the cost curve *C*(*Q*) is relatively flat at the bottom. Hence, being off target doesn't have a large impact on total cost. Being off the optimal order quantity in terms of ordering too much has a slow rate of growth in cost, whereas, ordering too little can increase total cost dramatically at some point. As Q increases, the component of *Ihc* that is increasing is only the cycle stock, not the safety stock or the in-transit stock. Furthermore, *Ihc* increases linearly. However, as Q decreases, $\left(\frac{D}{Q}\right)$ increases at an increasing rate.

The following example is important because it illustrates much of what has been talked about in this chapter, but it also shows an important caveat when using the EOQ or other optimization models. It also brings transportation costs into the discussion. So, the following example is not just an illustration of how to use what we have learned, it actually contains new material that is easiest to explain in the context of an example.

A regional retailer, Value Dime and Five has one distribution center that serves 500 stores. It only sells one SKU of toilet paper, its private label (Hunter TP) 2 ply extra coarse toilet paper with 543 sheets per roll (white with bold rough embossing). It replenishes stores in case pack quantities, and each case contains 80 rolls. Value Dime and Five only buys it by the truckload, which holds 560 cases, and pays $40 per case. Demand during lead time is approximately normally distributed with a mean of 80 cases and a standard deviation (σ_{DDLT}) of 30 cases. Since the lead time is usually a day and average daily demand faced by the distribution center is 80, Value Dime and Five orders when inventory position is 100. This might seem strange, but the company has a good reason for it. Stores typically have plenty of inventory, so even if Value Dime and Five is late on filling an order from a store, the stores usually don't run out of stock. Any orders that can't be filled when received from a store are filled later, as soon as the inventory is available in the distribution center. The transportation cost per truckload is $400. The owner of Value Dime and Five recognizes that transportation costs are high, so from an efficiency perspective he wants to maximize truck utilization and he only buys full truckloads. Based on his weighted average cost of capital and damage analysis, he estimates his inventory carrying cost factor to be about 0.25 of the value of inventory per year. He estimates all other costs of ordering associated with purchasing, accounts payable, receiving, and so on to be about $50 per order. For each case that is back-ordered by a store because the distribution center doesn't have it in stock, the cost is about $5 per case and is the result of administrative workarounds. The terms of sale are FOB Origin, Freight Prepaid because Value Dime and Five has a much larger transportation spend, allowing it to get better deals on transportation rates. Since it is FOB Origin, Value Dime and Five owns the inventory in transit. Value Dime and Five estimates its in-transit inventory carrying cost factor to be about 0.23 of the value of inventory per year. Value Dime and Five has a new analyst who is pushing the idea that the company should not focus on maximizing transportation utilization, but rather should focus on minimizing total cost. The analyst is proposing the company use the economic order quantity model in determining how much to order at a time rather than just ordering a truckload. Value Dime and Five is open every day of the year.