CD-ROM INCLUDED

Basic Principles and Calculations in Chemical Engineering

Eighth Edition

David M. Himmelblau • James B. Riggs

International Series in the
Physical and Chemical Engineering Sciences

Basic Principles and Calculations in Chemical Engineering

Eighth Edition

6.2 Sequential Multi-Unit Systems

Let's first examine a multi-unit system composed of a **sequential combination of units**. Figure 6.3a illustrates a sequential combination of mixing and splitting stages. Streams 1 and 2 combine to form the first mixing point, streams 3 and 4 also combine in the box for the second mixing point, and stream 5 splits (at presumably a pipe junction) into streams 6 and 7. Note that the first mixing point occurs at the combination of streams 1 and 2 (presumably a junction of pipes) while the second mixing occurs where streams 3 and 4 enter the box (presumably representing a process). You will encounter both types of mixing in the problems and examples in this book and in flowsheets in professional practice.

Examine Figure 6.3a. Which streams must have the same composition? Do streams 5, 6, and 7 have the same composition? Yes, because streams 6 and 7 flow from a splitter. Do streams 3, 4, and 5 have the same composition? It's quite unlikely. Stream 5 is some type of average of the compositions of streams 3 and 4 (in the absence of reaction). What is the composition inside the system (the box)? It will have the same composition as stream 5 only if streams 3 and 4 are really **well mixed** in the subsystem represented by the box.

How many material balances can you formulate for the system and subsystems shown in Figure 6.3a? First, let's examine how many total material balances you can write. You can write an **overall total material balance**, namely, a total balance on the system that includes all of the three subsystems within the overall system boundary, which is denoted by the dashed line labeled I in Figure 6.3b.

In addition, you can make a total balance on each of the three subsystems that make up the overall system, as denoted by the boundaries

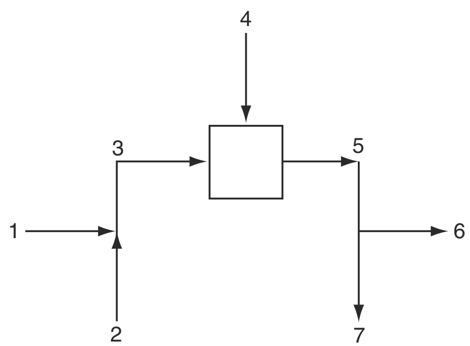

Figure 6.3a Serial mixing and splitting in a process without reaction

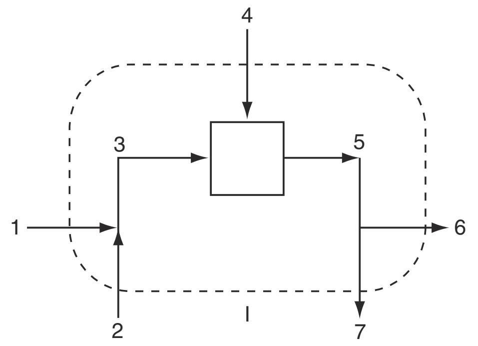

Figure 6.3b The dashed line I designates the boundary for an overall total material balance made on the system in Figure 6.3a.

indicated by the dashed lines II, III, and IV in Figure 6.3c. Finally, you can make a balance about each of the combinations of two subsystems as indicated by the dashed-line boundaries V and VI in Figures 6.3d and 6.3e, respectively.

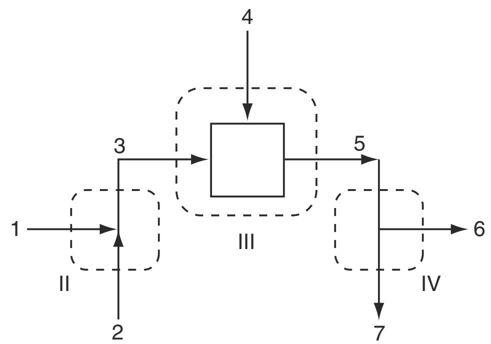

Figure 6.3c Dashed lines II, III, and IV denote the boundaries for material balances around each of the individual units constituting the overall process.

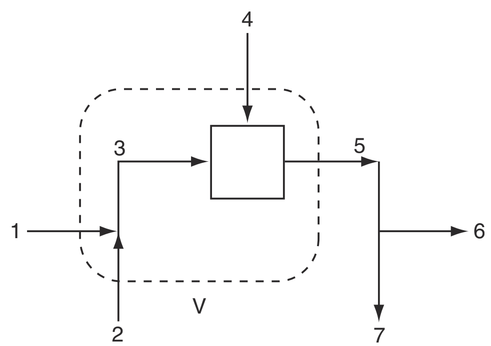

Figure 6.3d The dashed line V denotes the boundary for material balances around a subsystem composed of the first mixing point plus the subsystem portrayed by the box.

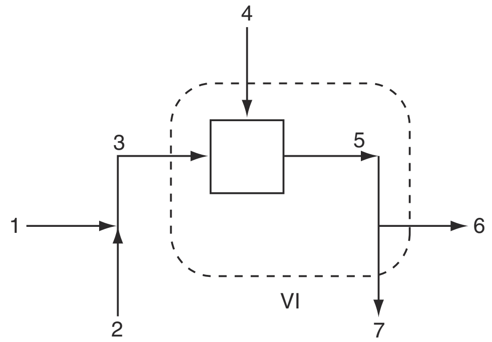

Figure 6.3e The dashed line VI denotes the boundary for material balances about a subsystem composed of the process portrayed by the box plus the splitter.

You can conclude for the system shown in Figure 6.3a that you can make total material balances on six different combinations of subsystems.

The important question is: How many independent material balance equations can be written for the process illustrated in Figure 6.3a if more than one component exists? In Section 6.1 we stated that you can write one independent equation for each component in each subsystem except for the splitter, for which you can write only one independent material balance equation. For Figure 6.3a, assume that three components are present in each of the separate subsystems shown in Figure 6.3c. You can write three independent material balance equations about the first pipe junction, three for the box, plus one independent equation for the splitter, for a total of seven (7) independent material balance equations. How many material balances are possible if you include *all of the redundant equations*, total as well as component balances?

Figure 6.3b:	3 components plus 1 total
Figure 6.3c:	$3 \times 3 = 9$ components plus 3 total
Figure 6.3d:	3 components plus 1 total
Figure 6.3e:	3 components plus 1 total

The total is 24. Which 7 of the 24 equations would be the most appropriate to choose to retain independence and solve easily?

Be careful to **select an independent set of equations**. As an example of what not to do, do not select three component balances for (a) the first pipe junction shown in Figure 6.3c, (b) the box, and (c) one balance for the splitter plus an overall total balance (Figure 6.3b). This set of equations would not be independent because, as you know, the overall balance is just the sum of the respective species balances for the individual unit. However, the total balance could be substituted for one of the component balances.

What strategy should you use to select the particular unit or subsystem with which to start formulating your independent equations for a process composed of a sequence of connected units? A good, but time-consuming, way to decide is to determine the degrees of freedom for various subsystems (single units or combinations of units) selected by inspection. A subsystem with zero degrees of freedom is a good starting point. Frequently, the best way to start is to make material balances for the **overall process**, ignoring information about the internal **connections**. If you ignore all of the internal streams and variables within a set of connected subsystems, you can treat the overall system exactly as you treated a single system in Chapters 3 through 5.

Example 6.1 Determination of the Number of Independent Material Balances in a Process with Multiple Units

Lactic acid ($C_3H_6O_3$) produced by fermentation is used in the food, chemical, and pharmaceutical industries. Figure E6.1 illustrates the mixing of components to form a suitable fermentation broth. The whole system is steady-state and open. The arrows designate the direction of the flows. No reaction occurs in any of the subsystems.

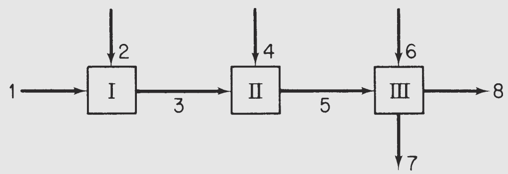

Figure E6.1

The mass compositions of each stream are as follows:

1. Water (W): 100%
2. Glucose (G): 100%
3. W and B, concentrations known: $\omega_W = 0.800$ and $\omega_G = 0.200$
4. *Lactobacillus* (L): 100%
5. W, G, and L, concentrations known: $\omega_W = 0.769$, $\omega_G = 0.192$, $\omega_L = 0.0385$
6. Vitamin G with amino acids and phosphate (V): 100%
7. $\omega_W = 0.962$, $\omega_V = 0.0385$
8. $\omega_G = 0.833$, $\omega_L = 0.167$

What is the maximum number of independent mass balances that can be generated for this system?

Solution

From taking into account each of the three units as subsystems, you certainly can make nine component equations as follows: Let's bypass any total balance for any of the three units as well as any overall component or total balance, or any of the possible balances for combinations of units.

	Total Number of Component Balances
At unit I, two components are involved	2
At unit II, three components are involved	3
At unit III, four components are involved	4
Total	9

However, not all of the balances are independent. In the following list of the component balances, all of the known component concentrations have been inserted. F_i represents the stream flow designated by the subscript.

Subsystem I:

Component Balances
$$\text{A: } F_1(1.00) + F_2(0) = F_3(0.800) \qquad \text{(a)}$$
$$\text{B: } F_1(0) + F_2(1.00) = F_3(0.20) \qquad \text{(b)}$$

Subsystem II:

Component Balances
$$\text{A: } F_3(0.800) + F_4(0) = F_5(0.769) \qquad \text{(c)}$$
$$\text{B: } F_3(0.200) + F_4(0) = F_5(0.192) \qquad \text{(d)}$$
$$\text{C: } F_3(0) + F_4(1.00) = F_5(0.0385) \qquad \text{(e)}$$

Subsystem III:

Component Balances
$$\text{A: } F_5(0.769) + F_6(0) = F_7(0.962) + F_8(0) \qquad \text{(f)}$$
$$\text{B: } F_5(0.192) + F_6(0) = F_7(0) + F_8(0.833) \qquad \text{(g)}$$
$$\text{C: } F_5(0.0385) + F_6(0) = F_7(0) + F_8(0.167) \qquad \text{(h)}$$
$$\text{D: } F_5(0) + F_6(1.00) = F_7(0.086) + F_8(0) \qquad \text{(i)}$$

If you take as an arbitrary basis $F_1 = 100$, seven values of F_i are unknown; hence only seven independent equations need to be written. Can you recognize by inspection that among the entire set of nine equations, two are

(*Continues*)

Example 6.1 Determination of the Number of Independent Material Balances in a Process with Multiple Units *(Continued)*

indeed redundant, and hence a unique solution can be obtained using the seven independent equations?

If you solved the nine equations by hand *sequentially*, starting with Equation (a) and ending with Equation (i), along the way you would notice that Equation (d) is redundant with Equation (c), and Equation (h) is redundant with Equation (g). The redundancy of Equations (c) and (d) becomes apparent if you recall that the sum of the mass fractions in a stream is unity, hence an implicit relation exists between Equations (c) and (d) so they are not independent. Why are Equations (g) and (h) not independent?

As you inspect the set of Equations (a) through (i) with the viewpoint of solving them sequentially, you will note that each one can be solved for one variable. Look at the following list:

Equation	Determines	Equation	Determines
(a)	F_3	(f)	F_7
(b)	F_2	(g)	F_8
(c)	F_5	(h)	F_8
(d)	F_5	(i)	F_6
(e)	F_4		

If you entered Equations (a) through (i) into a software program that solves equations, you would receive an error notice of some type because the set of equations includes redundant equations.

If the fresh facts which come to our knowledge all fit themselves into the scheme, then our hypothesis may gradually become a solution.

Sherlock Holmes in Sir Arthur Conan Doyle's "The Adventure of Wisteria Lodge," in *The Complete Sherlock Holmes*

If you make one or more component mass balances around the combination of subsystems I plus II, or II plus III, or I plus III in Example 6.1, or around the entire set of three units, no additional *independent* mass balances will be generated. Can you substitute one of the indicated alternative mass balances for an independent species mass balance? In general, yes.

In calculating the degree-of-freedom analysis for problems involving multiple units, you must be careful to involve only independent material